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MATHÉMATIQUES

CORRECTION D'UNE DÉMONSTRATION

Par SERGE BERNSTEIN, de l'Académie

Entre autres propositions j'ai donné dans ma conférence «Sur les équations différentielles stochastiques» (1) le lemme III (p. 23): *Si l'équation*

$$\Delta Y_n = \Phi(Y_n, t_n, \sqrt{\Delta t_n}, \alpha_n)\sqrt{\Delta t_n} + Q(Y_n, t_n, \sqrt{\Delta t_n}, \alpha_n)\Delta t_n^{3/2} \qquad (27)$$

est équivalente à l'équation quasi-linéaire ou bornée

$$\Delta y_n = \Phi(y_n, t_n, \sqrt{\Delta t_n}, \alpha_n)\sqrt{\Delta t_n}, \qquad (25\text{ bis})$$

(où $|Q(y, t, \sqrt{\Delta t}, \alpha)| < \bar{Q}(L)$, *pour* $|y| \leqslant L$, $t \leqslant T$, $\bar{Q}(L)$ *étant une fonction finie donnée quelconque de* L*), la distribution des probabilités de* Y_n *pour* $t_n = t \leqslant T$ *tend pour* $\Delta t_i \to 0$ *vers la distribution limite de* y_n, *correspondant à la même distribution initiale.*

A cause de l'oubli d'un terme dans les inégalités qui servent de base à la démonstration de ce lemme, cette démonstration est insuffisante. Bien qu'au seul endroit, où ce lemme est utilisé dans la suite, son application est, en réalité, superflue, car l'équation considérée (29) (p. 25) est elle-même bornée, l'importance du lemme III lui-même m'oblige cependant d'en fournir ici une démonstration correcte. Il est nécessaire dans ce but d'étendre d'abord le lemme II du travail cité (p. 22) aux équations *équivalentes*:

Si l'équation (27) *est équivalente à l'équation*

$$\left.\begin{array}{c}\Delta y_n = \Phi(y_n, t_n, \sqrt{\Delta t_n}, \alpha_n)\sqrt{\Delta t} = A\Delta t_n + f\sqrt{\Delta t_n} \\ (E.M.f = 0,\ |\Phi'_y| < c_1)\end{array}\right\} \qquad (25)$$

quasi-linéaire ou satisfaisant seulement à l'inégalité

$$yA < c(1 + y^2), \qquad (26)$$

c *et* c_1 *étant des constantes, dans ces conditions, quelque petit que soit* $\varepsilon > 0$, *il est possible de fixer deux nombres* L *et* λ *tels que la probabilité des inégalités* $|Y_i| \leqslant L$ *pour toutes les valeurs* $i \leqslant n$, *correspondant aux subdivisions de* $t = t_n = \sum_0^{n-1} \Delta t_i$, *soit supérieure à* $1 - \varepsilon$, *dès que* $\Delta t_i < \lambda$.

En effectuant le changement des variables introduit à l'endroit cité

$$Z_n = \frac{1}{2}\log(Y_n^2 + 1),$$

on a

$$Z_{n+1}-Z_n=\frac{Y_n}{Y_n^2+1}\Phi\sqrt{\Delta t_n}+\frac{1-Y_n^2}{2(Y_n^2+1)^2}\Phi^2\Delta t_n+\Theta\rho(M)\,\Delta t_n^{3/2},$$

où $|\Theta|<1$, $\rho(M)$ étant une fonction croissante finie de $M=\max|Z_n|$. L'énoncé actuel du lemme II se rapportant à toutes les équations (27) *équivalentes* à (25), nous ne pouvons plus affirmer que $\rho(M)$ *est borné*, ce qui était légitime à l'endroit cité, car il n'y était question que de l'équation (25) elle-même. A cause de cela, en reproduisant la marche de l'ancienne démonstration, nous indiquerons seulement les modifications que nécessite notre extension actuelle. Ainsi, par addition, nous aurons pour toute valeur de $i\leqslant n$

$$Z_{i+1}<Z_0+c_2t_{i+1}+\sum_0^i\xi_k\sqrt{\Delta t_k}+t\rho(M_i)\sqrt{\Delta t}\,(E.\,M.\,\xi_k=0),$$

où c_2 est une constante, $M_i=\max\limits_{k\leqslant i}Z_k$, $\Delta t\geqslant\Delta t_i\,(i\leqslant n)$. Donc, la probabilité de la réalisation simultanée de toutes les inégalités

$$Z_i<Z_0+c_2t+\frac{1}{\varepsilon}\sqrt{c_3t}+t\rho(M_n)\sqrt{\Delta t}\;\;(i\leqslant n)$$

où c_3 est une constante, sera supérieure à $1-\varepsilon$. Posons $Z_0+c_2t+\frac{1}{\varepsilon}\sqrt{c_3t}=R$ et supposons $\Delta t<\lambda$, où $\sqrt{\lambda}\leqslant\frac{R}{t\rho(3R)}$; nous aurons a fortiori $Z_i<2R$. Par conséquent, la probabilité des inégalités simultanées

$$|Y_i|<e^{2R}=L\;\;(i\leqslant n)$$

sera également supérieure à $1-\varepsilon$.

A présent la démonstration du lemme III peut être faite, comme il suit, même dans l'hypothèse plus large $A'_y<c$ [qui entraîne (26)] au lieu de $|A'_y|<c$ correspondant à la quasi-linéarité. Il suffit d'appliquer le raisonnement qui nous a servi à établir le lemme IV (p. 30). Posons à cet effet $u_n=Y_n-y_n$ et désignons par $\mathfrak{M}_L\psi(Y_n,y_n,\alpha_n)$ l'espérance mathématique de toute fonction $\psi(Y_n,y_n,\alpha_n)$ calculée dans l'hypothèse que $|Y_i|\leqslant L$, $|y_i|\leqslant L\,(i\leqslant n)$. On a

$$u_{n+1}-u_n=[\Phi(Y_n,t_n,\sqrt{\Delta t_n},\alpha_n)-$$

$$-\Phi(y_n,t_n,\sqrt{\Delta t_n},\alpha_n)]\sqrt{\Delta t_n}+Q(Y_n,t_n,\sqrt{\Delta t_n},\alpha_n)\,\Delta t_n^{3/2},$$

d'où

$$\mathfrak{M}_L u_{n+1}^2\leqslant\mathfrak{M}_L[u_n+(\Phi(Y_n,t_n,\sqrt{\Delta t_n},\alpha_n)-$$

$$-\Phi(y_n,t_n,\sqrt{\Delta t_n},\alpha_n))\sqrt{\Delta t_n}+Q\Delta t_n^{3/2}]^2<\mathfrak{M}_L[u_n+$$

$$+(\Phi(Y_n,t_n,\sqrt{\Delta t_n},\alpha_n)-\Phi(y_n,t_n,\sqrt{\Delta t_n},\alpha_n))\sqrt{\Delta t_n}]^2+P(L)\,\Delta t_n^{3/2},$$

$P(L)$ représentant une fonction déterminée finie de L. Par conséquent, en observant que $\mathfrak{M}_L f=0$, on aura

$$\mathfrak{M}_L u_{n+1}^2<\mathfrak{M}_L u_n^2+2\mathfrak{M}_L u_n[A(Y_n,t_n,\sqrt{\Delta t_n},\alpha_n)-$$

$$-A(y_n,t_n,\sqrt{\Delta t_n},\alpha_n)]\,\Delta t_n+\mathfrak{M}_L[\Phi(Y_n,t_n,\sqrt{\Delta t_n},\alpha_n)-$$

$$-\Phi(y_n,t_n,\sqrt{\Delta t_n},\alpha_n)]^2\,\Delta t_n+P(L)\,\Delta t_n^{3/2}<\mathfrak{M}_L u_n^2(1+$$

$$+h\Delta t_n)+P(L)\,\Delta t_n^{3/2}\leqslant\mathfrak{M}_L u_n^2(1+h\Delta t_n)+h\delta\Delta t_n,$$

où $h > 1$ est une constante, $\delta = \frac{P(L)}{h}\sqrt{\mu}$, $\Delta t_n \leqslant \mu$. Par conséquent (à cause de $u_0 = 0$), on a

$$\mathfrak{M}_L u_{n+1}^2 < (\mathfrak{M}_L u_n^2 + \delta)(1 + h\Delta t_n) - \delta =$$

$$= \delta[(1 + h\Delta t_0)\ldots(1 + h\Delta t_n) - 1] < \delta[e^{ht} - 1] < P(L)\, e^{ht}\sqrt{\mu}.$$

Donc, si $\mu = \lambda$ est assez petit, pour que $P(L)\, e^{ht}\sqrt{\mu} < \varepsilon\beta^2$, la probabilité de l'inégalité

$$|Y_{n+1} - y_{n+1}| < \beta, \qquad (I)$$

lorsqu'on sait que $|Y_i| \leqslant L$, $|y_i| \leqslant L\,(i \leqslant n)$, devient supérieure à $1 - \varepsilon$. Par conséquent, à cause du lemme II, la probabilité a priori de (I) est supérieure à $1 - 3\varepsilon$. Il y a lieu de noter que toutes nos conclusions subsistent, si dans la définition de *l'équivalence* de l'équation (27) on remplace le facteur $\Delta t^{3/2}$ du terme complémentaire par $\Delta t\varphi(\Delta t)$, où $\varphi(\Delta t) \to 0$ avec $\Delta t \to 0$.

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LITTÉRATURE CITÉE

[1] S. Bernstein, Equations différentielles stochastiques, Actualités Scientifiques, 738. Les fonctions aléatoires, Paris (1938).

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MATHEMATIK

ÜBER HALBUNITÄRE OPERATOREN

Von A. PLESSNER

(*Vorgelegt von A. N. Kolmogorov, Mitglied der Akademie, d. 27. X. 1939*)

Es sei H ein unitärer (komplexer Hilbertscher) Raum, der auch nichtseparabel sein kann. Ein Operator U in H heisst halbunitär, wenn für jedes h aus H $(Uh, Uh) = (h, h)$, d. h. $U^*U = E$ gilt (unitär, wenn ausserdem $UU^* = E$). Im Anschluss an unsere Mitteilung (2) bestimmen wir sämtliche halbunitäre Operatoren, die bei vorgegebenem maximalen Operator A in der Form $F(A)$ darstellbar sind, wo $F(A)$ eine Funktion von A in engerem Sinne ist*. Wir bezeichnen mit $B(\sigma)$ eine Funktion, die fast überall mit $\lim_{\tau \to 0} B(\lambda)$, $\lambda = \sigma + i\tau$, übereinstimmt, und für die mit Ausnahme einer Menge von A-Masse Null $|B(\sigma)| = 1$ ist, mit $v_0(\sigma)$ eine reelle Funktion fast überall gleich $\lim_{\tau \to 0} v_0(\sigma, \tau)$, wo $B(\lambda)$ und $v_0(\sigma, \tau)$ die in (2) angegebene Bedeutung haben. Es gilt dann der

Satz I. *Ein halbunitärer Operator U, der in der Form $F(A)$ darstellbar ist, wo A ein vorgegebener maximaler Operator und $F(A)$ eine Funktion von A im engeren Sinne ist, wird durch die Funktionen:*

$$F_1(\sigma) = e^{\varepsilon i t \sigma}; \quad F_2(\sigma) = B(\sigma); \quad F_3(\sigma) = e^{i v_0(\sigma)}$$

und deren Produkte bestimmt, wobei $t \geqslant 0$ ist.

Die Operatoren $U_t = e^{\varepsilon i t A}$ bilden also eine Schaar halbunitärer Operatoren, für welche:

1) $U_{s+t} = U_s U_t$, wenn $s \geqslant 0$, $t \geqslant 0$

2) $(U_t h, g)$ stetig in t ist.

Allgemein nennen wir eine Schaar von halbunitären Operatoren U_t, $t \geqslant 0$, mit den Eigenschaften 1) und 2) eine stetige eingliedrige Halbgruppe halbunitärer Operatoren. Die zum maximalen A gehörige Halbgruppe $e^{\varepsilon i t A}$ bestimmt A eindeutig. Es gilt aber auch

Satz II. *Ist U_t eine stetige eingliedrige Halbgruppe halbunitärer Operatoren, so gibt es einen maximalen Operator A, dessen Resolvente in der unteren Halbebene existiert, so dass*

$$U_t f = e^{itA} f = \int_{-\infty}^{\infty} e^{it\xi} dE(\Delta_\xi) f \tag{1}$$

wo $E(\Delta)$ die Spektralfunktion von A bedeutet (1). Es genügt übrigens an Stelle der Stetigkeit nur die Messbarkeit von $(U_t h, g)$ vorauszusetzen.

* Für Bezeichnungen vgl. (2).

Ist U halbunitär, so hat man in U^n $n \geqslant 0$ eine diskrete zyklische Halbgruppe halbunitärer Operatoren, für die eine zu (1) analoge Integraldarstellung gilt.

Satz III. *Ist U ein halbunitärer Operator, so gilt die Integraldarstellung*

$$U^n f = \int_{-\pi}^{\pi} e^{in\xi} dI(\delta_\xi) f; \tag{2}$$

hier bedeutet $I(\delta)$ eine additive Funktion des Intervalls δ auf der Kreisperipherie $|\lambda| = 1$, für deren Diskrepanz $D(\delta_1, \delta_2) = I(\delta_1 \delta_2) - I(\delta_2) I(\delta_1)$

$$(D(\delta_1, \delta_2) h_1 h) = \int_{\delta_1} \varphi(e^{i\xi}, \delta_2; h)\, de^{i\xi} = \int_{\delta_2} \overline{\psi(\delta_1, e^{i\xi}; h)}\, de^{-i\xi}$$

gilt, wo $\varphi(e^{i\vartheta}, \delta_2; h)$ und $\psi(\delta_1, e^{i\Theta}; h)$ Randwerte analytischer im Einheitskreise $|\lambda| < 1$ Funktionen $\varphi(\lambda, \delta_2; h)$ und $\psi(\delta_1, \lambda; h)$, $\lambda = re^{i\Theta}$ sind, für die

$$\int_{-\pi}^{\pi} |\varphi(\lambda, \delta_2; h)|\, d\vartheta < \varkappa_1(\delta_2; h); \int_{-\pi}^{\pi} |\psi(\delta_1, \lambda; h)|\, d\vartheta < \varkappa_2(\delta_1, h).$$

Umgekehrt erzeugt eine additive Intervallfunktion $I(\delta)$ auf $|\lambda| = 1$ mit diesen Diskrepanzeigenschaften und der Vollständigkeit $\int_{-\pi}^{\pi} dI(\delta_\xi) f = f$ gemäss (2) eine diskrete zyklische Halbgruppe halbunitärer Operatoren.

Der Satz III steht in engem Zusammenhang mit der in [1] entwickelten Spektraltheorie maximaler Operatoren, gestattet aber eine independente Ableitung. Es sei $R_\mu = U(-\mu E)^{-1}$ die Resolvente von U für $|\mu| > 1$ und für $|\lambda| < 1$, $R_\lambda = R^*_{\lambda'}$, wo $\lambda' = \frac{1}{\bar{\lambda}}$. Es gilt dann die Funktionalgleichung

$$\left(\frac{\mu}{\lambda} - 1\right) R_\lambda R_\mu + \frac{R_\lambda}{\lambda} + \mu R_\mu + E = 0$$

und wenn noch $\left(\lambda' = \frac{1}{\bar{\lambda}}\right)$

$$Q(r, \vartheta) = -\frac{1}{2\pi}\left(\frac{R_\lambda}{\lambda} + \frac{R_{\lambda'}}{\bar{\lambda}} + E\right); \quad \lambda = re^{i\vartheta}$$

gesetzt wird, so hat man

$$(Q(r, \vartheta) h, h) \geqslant 0, \int_{-\pi}^{\pi} (Q(r, \vartheta) h, h)\, d\vartheta = (h, h)$$

und $I(\delta)$ in (2) bestimmt sich aus

$$(I(\delta) h, h) = \lim_{r \to 1} \int_{\delta} (Q(r, \vartheta) h, h)\, d\vartheta$$

[δ — Stetigkeitsintervall von $(I(\delta) h, h)$].

Betrachtet man nun den Teilraum $H^{(0)}$ der Elemente g, für die $U^* g = 0$ gilt, und wird $H^{(0)}$ aufgespannt vom (unter Umständen nicht abzählbaren) normierten Orthogonalsystem $\{g_\alpha\}$, so wird durch die Elemente $I(\delta) g_\alpha$ bei veränderlichem δ ein invarianter Teilraum H_α für U erzeugt. Die Gesamtheit der zu sämtlichen H_α, die zueinander orthogonal sind, orthogonalen Elemente bildet ebenfalls einen invarianten Teilraum H_0, auf dem U unitär ist. Auf jedem der H_α ist U einem Operator V isometrisch äquiva-

lent, dessen Definition wir gleich angeben. Es sei L die Funktionengesamtheit der $F(e^{i\theta})$, die fast überall gleich $\lim_{r\to 1} F(\lambda)$, $\lambda = re^{i\theta}$, wo $F(\lambda)$ eine im Einheitskreise analytische Funktion mit

$$\int_{-\pi}^{\pi} |F(re^{i\theta})|^2 d\theta < K$$

ist. L wird vermöge der Definition des skalaren Produkts

$$(F_1, F_2) = \frac{1}{2\pi} \int_{-\pi}^{\pi} F_1(e^{i\theta}) \overline{F_2(e^{i\theta})}\, d\theta$$

ein unitärer Raum. In L ist der Operator V durch

$$VF(e^{i\theta}) = e^{i\theta} F(e^{i\theta})$$

definiert. H_a ist nun isometrisch auf L abbildbar, so dass U in L den Operator V induziert. Wir bemerken noch, dass wenn $C(e^{i\theta}) = \lim_{r\to 1} C(\lambda)$, wo $C(\lambda)$ ein im Einheitskreise beschränkte analytische Funktion bedeutet, der Operator

$$C(U) = \int_{-\pi}^{\pi} C(e^{i\xi})\, dI(\delta_\xi)$$

betrachtet auf H_a in L den Operator $C(V)$

$$C(V) F(e^{i\theta}) = C(e^{i\theta}) F(e^{i\theta})$$

induziert.

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ZITIERTE LITERATUR

[1] A. Plessner, C. R. Acad. Sci. URSS, XXII, No. 5 (1939). [2] A. Plessner, ibid., XXIII, No. 4 (1939).

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MATHEMATICS

ON ONE-PARAMETRICAL GROUPS OF OPERATORS IN A NORMED SPACE

By I. GELFAND

(Communicated by A. N. Kolmogorov, Member of the Academy, 3. XI. 1939)

1. Suppose we are given a family of operators U_t in a linear space E (t — a real number) satisfying the following conditions:

1°. U_t is a weakly continuous function of t, i. e. $(U_t x, f)$ is for every x and f a continuous function of t; (y, f) or (f, y) means the value of the linear functional f at the point y of the space E*.

2°. $U_{t+s} = U_t \cdot U_s$ for arbitrary numbers t and s and $U_0 = 1$.

3°. $|U_t| \leqslant M$ (in the sequel we shall for simplicity assume that $M = 1$. This can be always achieved by replacing the norm in the space by $|x|_1 = \sup_t |U_t x|$ which is equivalent to it).

We shall prove that such a family U_t of operators may be represented in the form $U_t = e^{tA}$ and shall study the properties of the operator A (the operator A will be, in general, an unbounded operator). In the case, when E is a Hilbert space and U_t is a family of unitarian operators, this theorem was proved by Stone (1). The problem which is solved in the present paper was proposed by A. N. Kolmogorov.

2. Construction of the Operator A. It is natural to construct the operator A in the following way. Suppose that for some x there exists

$$\lim_{h \to 0} \frac{U_h x - x}{h}.$$

Then put

$$Ax = \lim_{h \to 0} \frac{U_h x - x}{h}. \tag{1}$$

Let us prove that the limit (1) exists for an everywhere dense set of elements x of the space E.

Consider the following operator:

$$C = \int_{-\infty}^{+\infty} U_s \, d\varphi(s), \tag{2}$$

* See § 5, where condition 1° is weakened.

where $\varphi(s)$ is a function of bounded variation on the real axis. The equality (2) has the following sense: for every x and f (f is a functional, x an element of the space E)

$$(Cy, f) = \int_{-\infty}^{+\infty} (U_s\, y, f)\, d\varphi(s).$$

The existence of the operator C in the case when E is a separable or a regular space is proved in a previous paper of the author (2). The proof of the existence of the operator C in the general case will be given elsewhere.

It is easily seen that $|C| \leqslant \operatorname{var}_s [\varphi(s)]$ and

$$\frac{U_h - 1}{h} C = \int_{-\infty}^{+\infty} U_s d \frac{\varphi(s-h) - \varphi(s)}{h}.$$

We choose $\varphi(s)$ differentiable and such that

$$\lim_{h \to 0} \operatorname{var}_s \left[\frac{\varphi(s-h) - \varphi(s)}{h} - \varphi'(s) \right] = 0. \tag{3}$$

Then there exists $\lim\limits_{h \to 0} \frac{U_h - 1}{h} C$ (the limit being understood here in the sence of the convergence of operators by the norm) and is equal to

$$C_1 = \int_{-\infty}^{+\infty} U_s d\varphi'(s).$$

In fact,

$$\left| \frac{U_h - 1}{h} C - C_1 \right| = \left| \int_{-\infty}^{+\infty} U_s d \left[\frac{\varphi(s-h) - \varphi(s)}{h} - \varphi'(s) \right] \right| \leqslant$$

$$\leqslant \operatorname{var}_s \left[\frac{\varphi(s-h) - \varphi(s)}{h} - \varphi'(s) \right].$$

Consequently, for an arbitrary element

$$x = Cy, \tag{4}$$

the limit of the expression $\frac{U_h x - x}{h}$ exists (by the limit is meant here the strong limit).

We collect all elements x which may be obtained from the formula (4) with at least one admittable C [i. e. a C defined by the formula (2) with a function $\varphi(s)$ satisfying the equality (3)]. We shall prove that the set of these x is everywhere dense in E.

Assume the contrary: then by Hahn's theorem there exists a linear functional $f \neq 0$ such that

$$\int_{-\infty}^{+\infty} (U_t y, f)\, d\varphi(t) = 0$$

for any y and any function $\varphi(t)$ satisfying the relation (3). Consequently, $(U_t y, f) = 0$ for any y and t; in particular, putting $t = 0$, we have $(y, f) = 0$ for any y, i. e. $f = 0$, and we obtain a contradiction.

Thus, we have proved that *on an everywhere dense set of elements x there exists a limit in the sense of strong convergence of the expression* $\frac{U_h x - x}{h}$ *for* $h \to 0$. *Put*

$$Ax = \lim_{h \to 0} \frac{U_h x - x}{h}$$

for all those x, for which the limit of the right-hand side exists, be it even in the sense of weak convergence.

3. *Investigation of the Properties of the Operator A.* We prove in the first place that *the operator A is closed.* Before we proceed to the proof, we make a simple remark which will be used in the sequel.

If Ax is defined, then the expression $AU_t x$ is also defined and we have the equality

$$AU_t x = U_t Ax = \frac{d}{dt} U_t x. \tag{5}$$

This follows directly from the definition of A, if we observe that

$$\frac{U_{t+h} x - U_t x}{h} = U_t \frac{U_h x - x}{h} = \frac{U_h [U_t x] - U_t x}{h}.$$

We pass now to the proof of the closedness of the operator A.

An operator is said to be closed, if $x_n \to x$ and $Ax_n \to y$ imply that Ax is defined and that $y = Ax$.

Let $x_n \to x$ and $Ax_n \to y$. According to (5)

$$U_t x_n - U_0 x_n = \int_0^t \frac{d}{ds} U_s x_n \, ds = \int_0^t U_s Ax_n \, ds.$$

Letting $n \to \infty$, we obtain

$$U_t x - x = \int_0^t U_s y \, ds$$

(we have the right to pass to the limit under the sign of the integral, since $|U_s Ax_n - U_s y| \leqslant |Ax_n - y|$), or

$$\frac{U_t x - x}{t} = \frac{1}{t} \int_0^t U_s y \, ds.$$

For $t \to 0$ the weak limit of the right-hand side exists and is equal to y. Consequently, Ax exists and $Ax = y$, q. e. d.

Let us now show that the spectrum of the operator A is situated on the imaginary axis. To this end we prove first the following

Lemma 1.

$$|(A - \lambda 1) x| \geqslant |\sigma| \cdot |x|, \tag{6}$$

where $\lambda = \sigma + i\tau$. In the general case this estimation cannot be improved.

Proof. Suppose first that $\tau = 0$. It is evidently sufficient to prove our assertion for $|x| = 1$. Suppose that for some x with $|x| = 1$ (6) is not satisfied, i. e. that $|(A - \lambda 1) x| \leqslant a < |\sigma|$. According to (5)

$$\frac{d}{dt} U_t x = U_t Ax = U_t (A - \sigma 1) x + \sigma U_t x. \tag{7}$$

There exists an f, $|f| = 1$, such that $(f, x) = 1$. Applying f to both sides of (7) and putting $(U_t x, f) = \varphi(t)$, we obtain

$$\frac{d}{dt} \varphi(t) = \sigma \varphi(t) + \psi(t)$$

where $\psi(t)=(U_t(A-\sigma 1)x, f)$, and consequently, $|\psi(t)|\leqslant a<|\sigma|$. Thus, we have $|\varphi'(t)-\sigma\varphi(t)|\leqslant a<|\sigma|$ and $\varphi(0)=(U_0x, f)=1$. Let $\varphi(t)=\varphi_1(t)+i\varphi_2(t)$. Then obviously

$$|\varphi_1'(t)-\sigma\varphi_1(t)|\leqslant a<|\sigma| \quad \text{and} \quad \varphi_1(0)=\varphi(0)=1.$$

Let $\sigma>0$, $\varphi_1'(t)\geqslant\sigma\varphi_1(t)-a$; put $\sigma\varphi_1-a=\rho(t)$. Then

$$\rho'(t)\geqslant\sigma\rho(t) \quad \text{and} \quad \rho(0)=\sigma-a>0. \tag{8}$$

The function $\rho(t)$ remains positive on the interval $(0, \infty)$. In fact, let t_0 be the leftmost of the points of the interval $(0, \infty)$ at which $\rho(t_0)=0$. Then $\rho(t_0)-\rho(0)=t_0\rho'(t_1)$, where $0<t_1<t_0$, and consequently $\rho'(t_1)<0$; on the other hand, $\rho'(t_1)\geqslant\sigma\rho(t_1)>0$. Thus, there exists no point on $(0, \infty)$ at which $\rho(t)$ vanishes, and consequently $\rho(t)$ does not change its sign. From (8) follows $\frac{\rho'(t)}{\rho(t)}\geqslant\sigma$, i. e. $\rho(t)\geqslant\rho(0)e^{\sigma t}$; consequently, $\rho(t)$, and therefore also $\varphi(t)=(U_tx, f)$, is unbounded. But this contradicts to the boundedness of $|U_t|$. Thus, for $\tau=0$ the lemma is proved.

If $\lambda=\sigma+i\tau$, we put $U_t^{(1)}=e^{-i\tau t}U_t$. Then

$$A^{(1)}x=\lim_{t\to 0}\frac{e^{-i\tau t}U_tx-e^{-i\tau t}x}{t}=Ax-i\tau x.$$

Applying the already obtained result to $A^{(1)}x$, we obtain

$$|Ax-\lambda x|\geqslant\sigma\cdot|x|.$$

Denote by Ω_A the domain of definition of the operator A.

Lemma 2. *$A-\lambda 1$ ($\lambda=\sigma+i\tau$, $\sigma\neq 0$) maps Ω_A on the whole space.*

Proof. We prove first that the aggregate of elements $Ax-\lambda x$ ($x\in\Omega_A$) is everywhere dense in E. Assume the contrary. Then there exists $f\neq 0$ such that $(Ax-\lambda x, f)=0$ for any $x\in\Omega_A$, i. e. $(Ax, f)=\lambda(x, f)$ for $x\in\Omega_A$. Since together with x also $U_tx\in\Omega_A$, $(AU_tx, f)=\lambda(U_tx, f)$, i. e. $\frac{d}{dt}(U_tx, f)=\lambda(U_sx, f)$. Consequently, $(U_tx, f)=(x, f)e^{\lambda t}$; since $e^{\lambda t}$ is unbounded on the real axis and (U_tx, f) is bounded, $(x, f)=0$ for $x\in\Omega_A$, i. e. $f=0$, and we obtain a contradiction.

We prove now that the set $(A-\lambda 1)x$, $x\in\Omega_A$, is closed. Let $y_n=Ax_n-\lambda x_n$ and $y_n\to y$. According to Lemma 1, the x_n also have a limit: $\lim x_n=x$. Since $x_n\to x$ and $Ax_n-\lambda x_n\to y$, $Ax_n\to\lambda x+y$, and since A is a closed operator, Ax is defined and $Ax=\lambda x+y$. Thus, $y=(A-\lambda 1)x$ belongs also to our set, i. e. we have proved its closedness.

From Lemmas 1 and 2 follows that the spectrum of the operator A is situated on the imaginary axis and that it is impossible to extend the domain of its definition without the set of its spectral points becoming the whole plane.

4. In the present paragraph we prove that for any h we have $e^{Ah}=U_h$; e^{hA} we define as follows:

$$e^{hA}x=\sum_{n=0}^{\infty}\frac{h^n}{n!}A^nx. \tag{9}$$

Let us show that the right-hand side of (9) has sense for an everywhere dense set of elements x.

Consider the aggregate of infinitely differentiable functions $\varphi(t)$ such that

$$\lim_{N\to\infty}\operatorname{var}[\varphi(t+h)-\varphi_N(t)]=0, \tag{10}$$

where

$$\varphi_N(t) = \sum_{n=0}^{N-1} \varphi^{(n)}(t)\,\frac{h^n}{n!}$$

(such will be, for instance, all functions of the form $\frac{P(t)}{a^2+t^2}$ where $P(t)$ is a trigonometrical polynomial, $a > |h|$). Then, for $x = \int_{-\infty}^{+\infty} U_t y\, d\varphi(t)$, $A^n x$ are defined for every n (the proof is obtained in the following way: we apply the arguments of § 2 to $Ax = \int_{-\infty}^{+\infty} U_t y\, d\varphi'(t)$, which proves the existence of $A^2 x$, etc.). In the same way as in § 2 we prove that the set of these x is everywhere dense in E. The series $\sum_{n=0}^{\infty} \frac{A^n x}{n!} h^n$ converges. In fact, $\sum_{n=0}^{N-1} \frac{A^n x}{n!} h^n = \int_{-\infty}^{+\infty} U_t y\, d\varphi_N(t)$ and consequently, according to (10), for $N \to \infty$ it converges to $\int_{-\infty}^{+\infty} U_t y\, d\varphi(t+h)$.

Let us now prove that if for any x the series $\sum_{n=0}^{\infty} \frac{A^n x}{n!} h^n$ converges, then its sum is equal to $U_h x$ [it is sufficient to demand that

$$\frac{|A^n x|}{n!} h_n \to 0; \tag{11}$$

as we shall see later, hereform already follows that the series (9) converges].

According to (5), § 3, $U_t A^n x = \frac{d^n}{dt^n} U_t x$. Consequently,

$$\left| U_{t+h}x - U_t \sum_{n=0}^{N-1} A^n x \frac{h^n}{n!} \right| = \left| U_{t+h}x - \sum_{n=0}^{N-1} \frac{d^n}{dt^n} U_t x \frac{h^n}{n!} \right| =$$

$$= \left| \frac{1}{N!} \int_t^{t+h} (t-s+h)^{N-1} \frac{d^N}{dt^N} U_s x\, ds \right|$$

(the last equality is a corollary of Taylor's formula with the integral form of the residual term), i. e., according to (11),

$$\left| U_{t+h}x - U_t \sum_{n=0}^{N-1} A^n x \frac{h^n}{n!} \right| = \left| \frac{1}{N!} \int_t^{t+h} (t-s+h)^{N-1} U_s A^N x\, ds \right| \leqslant$$

$$\leqslant \frac{|h|^N}{N!} |U_s A^N x| \leqslant \frac{|h|^N}{N!} |A^N x| \to 0. \tag{12}$$

Putting in (12) $t = 0$, we obtain:

$$U_h x = \sum_{n=0}^{\infty} A^n x \frac{h^n}{n!},$$

q. e. d.

5. In the present paragraph will be proved that from the weak measurability of the function U_t follows ist strong continuity.

Theorem. *Let U_t be weakly measurable, i. e. let $(U_t x, f)$ be a measurable function of t for any x and f. Then $U_t x$ is a strongly continuous function of t for every x, i. e. $t_n \to t$ implies $|U_{t_n} x - U_t x| \to 0$.*

Proof. In § 2 has been proved that $\lim_{h \to 0} \frac{U_h x - x}{h}$ exists for an everywhere dense set of elements x (this proof depended only on the weak measurability of U_t). Consequently, for elements of this everywhere dense set from $h \to 0$ follows $|U_h x - x| \to 0$, and consequently also from $t + h \to t$ follows $|U_{t+h} x - U_t x| \to 0$.

Let y be an arbitrary element. Then

$$|U_{t+h} y - U_t y| \leqslant |U_{t+h} y - U_{t+h} x| + |U_{t+h} x - U_t x| + |U_t x - U_t y| \leqslant$$
$$\leqslant 2|y - x| + |U_h x - x|.$$

Consequently, choosing x such that $|y - x| < \frac{\varepsilon}{4}$ and then δ such that for $|h| < \delta$, $|U_h x - x| < \frac{\varepsilon}{2}$, we obtain $|U_{t+h} y - U_t y| < \varepsilon$, q. e. d.

Corollary. If for $h \to 0$ the expression $\frac{U_h x - x}{h}$ weakly converges, then it converges also strongly.

Proof. Let

$$\lim \frac{U_h x - x}{h} = Ax,$$

$$\frac{U_h x - x}{h} = \frac{1}{h} \int_0^h U_t A x \, dt. \tag{13}$$

Since the expression under the sign of the integral is strongly continuous, for $h \to 0$ there exists a strong limit of the right-hand side, and consequently also of the left-hand side of formula (13), q. e. d.

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MATHÉMATIQUES

LES THÉORÈMES D'OSCILLATION POUR LES OPÉRATEURS LINÉAIRES DIFFÉRENTIELS D'ORDRE QUELCONQUE

Par M. KREIN

(Présenté par Serge Bernstein, de l'Académie, le 25. X. 1939)

Les résultats de notre Note précédente «Sur les fonctions de Green oscillatoires non-symétriques etc.» (1) permettent d'établir pour les opérateurs linéaires différentiels ordinaires d'ordre quelconque une suite de théorèmes qui sont analogues aux théorèmes classiques de Sturm pour les équations différentielles du deuxième ordre.

1. Soit

$$L(y)=\sum_{k=0}^{n} l_k(x)\frac{d^k y}{dx^k} \quad (n\geqslant 2)$$

un opérateur différentiel, dont les coefficients sont des fonctions continues dans un intervalle fermé $(a,\ b)$, et soit

$$l_n(x)>0 \text{ pour } a\leqslant x\leqslant b.$$

Supposons d'ailleurs que l'opérateur $L(y)$ admet dans l'intervalle fermé $(a,\ b)$ la représentation suivante:

$$L(y)=\rho_0\frac{d}{dx}\rho_1\frac{d}{dx}\rho_2\cdots\frac{d}{dx}\rho_n y \tag{1}$$

où les $\rho_k(x)\ (k=0,1,\ldots,n)$ sont des fonctions positives ayant k dérivées continues dans l'intervalle $(a,\ b)$, et que le système des conditions aux limites

$$\left.\begin{aligned} &y(a)=y'(a)=\ldots=y^{(p-1)}(a)=0\\ &y(b)=y'(b)=\ldots=y^{(q-1)}(b)=0\end{aligned}\right\}\quad (p+q=n,\ p,\ q>1) \tag{2}$$

n'est pas singulier par rapport à l'opérateur $L(y)$.

Nous avons démontré [voir (1), th. 4] que dans ce cas la fonction de Green $G(x,\ s)$ de l'opérateur $L(y)$, correspondant au système des conditions (2) étant multipliée par $(-1)^q$, devient un noyau oscillatoire.

Cela posé, $\rho(x)\ (a\leqslant x\leqslant b)$ étant une fonction sommable et presque partout positive, le système différentiel, formé de l'équation differentielle

$$L(y)-\lambda\rho y=0$$

et des conditions aux limites (2), n'a que des valeurs caractéristiques simples $\lambda_0, \lambda_1, \lambda_2, \ldots$, dont le signe coïncide avec celui de $(-1)^q$. Ainsi ces valeurs étant énumerées d'une manière convenable, on a

$$0 < (-1)^q \lambda_0 < (-1)^q \lambda_1 < (-1)^q \lambda_2 < \ldots$$

Théorème 1. *Soient*

$$\omega_1(x; \lambda),\ \omega_2(x; \lambda), \ldots, \omega_q(x; \lambda)\ (-\infty < \lambda < \infty)$$

des solutions linéairement indépendantes du système différentiel

$$L(y) - \lambda \rho y = 0;\ y(a) = y'(a) = \ldots = y^{(n-1)}(a) = 0.$$

Le déterminant de Wronski $W_q(x; \lambda) = W(\omega_1, \omega_2, \ldots \omega_q)$ *des fonctions de* x: $\omega_1(x; \lambda), \ldots, \omega_q(x; \lambda)$ *possède les propriétés suivantes:*

1°. *Pour* $-\infty < (-1)^q \lambda < (-1)^q \lambda_0$ *le déterminant* $W_q(x; \lambda)$, *considéré comme une fonction de* x, *n'a pas de zéros dans l'intervalle* $[a, b)$ *ouvert à gauche.*

2°. *Pour* $(-1)^q \lambda_{k-1} \leqslant (-1)^q \lambda < (-1)^q \lambda_k\ (k = 1,2, \ldots,)$ *le déterminant* $W_q(x; \lambda)$ *a dans l'intervalle* $[a, b)$ *exactement* k *zéros simples* $x_1(\lambda) < x_2(\lambda) < \ldots < x_k(\lambda)$.

3°. *Les zéros* $x_i(\lambda)$ $[(-1)^q \lambda_0 \leqslant (-1)^q \lambda;\ i = 1,2, \ldots]$ *sont des fonctions décroissantes de* $(-1)^q \lambda$ *et*

$$\lim_{(-1)^q \lambda \to \infty} x_i(\lambda) = a\ \ (i = 1,\ 2, \ldots).$$

On peut supposer, sans restreindre la généralité, que pour tout λ les fonctions $\omega_1, \omega_2, \ldots, \omega_q$ sont construites de telle manière que:

$$\omega_k^{p+j}(a;\ \lambda) = \omega_k^{p+j}(a;\ 0)\quad (j = 0,1, \ldots\ q-1;\ k = 1,\ 2, \ldots\ q).$$

Cela posé, on a la formule

$$W_q(\xi;\ \lambda) = W_q(\xi;\ 0) \prod_{k=0}^{\infty} \left(1 - \frac{\lambda}{\lambda_k(\xi)}\right)\quad (a < \xi \leqslant b), \tag{3}$$

où $\lambda_k(\xi)$ $(k = 0,\ 1,\ 2, \ldots)$ sont des valeurs caractéristiques du problème aux limites:

$$\left.\begin{aligned} &L(y) - \lambda \rho y = 0;\ y(a) = y'(a) = \ldots = y^{(p-1)}(a) = 0 \\ &y(\xi) = y'(\xi) = \ldots = y^{(q-1)}(\xi) = 0\ \ (a \leqslant x \leqslant \xi) \end{aligned}\right\} \tag{4}$$

On voit comme précédemment que

$$0 < (-1)^q \lambda_0(\xi) < (-1)^q \lambda_1(\xi) < (-1)^q \lambda_2(\xi) < \ldots$$

On peut aussi montrer que quand ξ décroît à partir de b jusqu'à a $(a < \xi \leqslant b)$, la fonction de ξ: $(-1)^q \lambda_k(\xi)$ $(k = 0,\ 1,\ 2, \ldots)$ croît de $(-1)^q \lambda_k$ $(\lambda_k = \lambda_k(b))$ jusqu'à ∞.

La dernière propriété des fonctions $(-1)^q \lambda_k(\xi)$ $(k = 0,\ 1,\ 2, \ldots)$ est presque triviale, si le problème aux limites (4) est autoadjoint. Mais dans les cas généraux que nous considérons sa démonstration exige des considérations assez délicates.

Le théorème 1 résulte immédiatement de la formule (3) et des propriétés mentionnées des fonctions $\lambda_k(\xi)$ $(k = 0,\ 1,\ 2, \ldots)$.

Théorème 2. *Le système différentiel*

$$\left.\begin{aligned} &L(y) - \lambda \rho y = 0;\ y(a) = y'(a) = \ldots = y^{(p-1)}(a) = 0,\ y^p(a) = 1 \\ &y(b) = y'(b) = \ldots = y^{(q-2)}(b) = 0\ ((-1)^q \lambda > 0) \end{aligned}\right\}$$

a une et une seule solution $\varphi_q(x;\lambda)$ *et celle-ci possède les propriétés suivantes:*

1°. *Pour* $0<(-1)^q\lambda<(-1)^q\lambda_0$ *la fonction* $\varphi_q(x;\lambda)\neq 0$ *à l'intérieur de l'intervalle* (a, b).

2°. *Pour* $(-1)^q\lambda_{k-1}\leqslant(-1)^q\lambda<(-1)^q\lambda_k$ $(k=1, 2, \ldots)$ *la fonction* $\varphi_q(x;\lambda)$ *a exactement* k *zéros simples à l'intérieur de l'intervalle* (a, b).

Le cas où $p=n-1$ et $q=1$ présente un intérêt spécial. On a alors

$$\varphi_1(x;\lambda)=\text{const}\ W_1(x;\lambda)$$

et, par suite, dans ce cas le théorème 1 contient le théorème 2. La fonction $\varphi_1(x;\lambda)$ étant la solution du système différentiel

$$L(y)=\lambda\rho y=0;\quad y(a)=y'(a)=\ldots=y^{(n-2)}(a)=0;\quad y^{(n-1)}(a)=1 \qquad (5)$$

on voit qu'on est arrivé à une généralisation des théorèmes classiques de Sturm se rapportant au cas particulier du système différentiel (5), lorsque $n=2$ et $L(y)\equiv L_2(y)\equiv\frac{d}{dx}\left(p\frac{dy}{dx}\right)-qy$ $(p(x)>0$ et $q(x)\geqslant 0$ pour $a\leqslant x\leqslant b)$.

Remarquons que l'opérateur de Sturm-Lionville lui aussi admet une représentation du type (1), à savoir

$$L_2(y)=\frac{1}{\psi}\frac{d}{dx}\left(p\psi^2\frac{d}{dx}\frac{y}{\psi}\right),$$

où $\psi(x)$ est une solution positive quelconque de l'équation homogène: $L_2(y)=0$.

Remarque. Les théorèmes 1 et 2 sont encore valables, lorsqu'on y remplace les conditions aux limites

$$y(a)=y'(a)=\ldots=y^{(p-1)}(a)=0$$

par un système quelconque de p conditions linéairement indépendantes

$$\sum_{k=0}^{n-1}\alpha_{ik}y^{(k)}(a)=0\quad(i=1, 2, \ldots, p)$$

pourvu que celui-ci possède la propriété suivante:

Les conditions aux limites

$$\sum_{k=0}^{n-1}\alpha_{ik}y^{(k)}(a)=0\quad(i=1, 2, \ldots, p)$$

$$y^{(i)}(b)=0\quad(i=0, 1, \ldots, q-1)$$

sont non-singulières, et la fonction de Green y correspondant étant multipliée par $(-1)^q$ est un noyau oscillatoire.

2. Nos résultats peuvent être étendus au cas des opérateurs quasi-différentiels. Soit, par exemple, $L(y)$ un opérateur quasi-différentiel donné dans l'intervalle fermé (a, b) par la formule (2), où les $\rho_k(x)$ $(k=0, 1, 2, \ldots, n)$ sont à présent des fonctions continues * quelconques généralement non-dérivables.

Posons pour abréger:

$$D^k y=\rho_{n-k}\frac{d}{dx}\rho_{n-k+1}\cdots\frac{d}{dx}\rho_n y\qquad(k=0, 1, \ldots, n)$$

* Cette condition peut aussi être affaiblie.

et désignons maintenant par $W(\varphi_1, \ldots, \varphi_k)$ $(k=1, 2, \ldots, n)$ le quasi-déterminant de Wronski, c'est-à-dire

$$W(\varphi_1, \ldots, \varphi_k) = | D^i \varphi_j | \; i=0,1, \ldots, k-1; \; j=1,2, \ldots, k).$$

Les théorèmes 1 et 2 sont aussi valables pour l'opérateur quasi-différentiel $L(y) = D^n y$, si l'on remplace partout dans ces théorèmes les dérivées ordinaires $y^{(k)}$ $(k=0, 1, \ldots, n)$ par les quasi-dérivées correspondantes $D^k y$ $(k=0, 1, \ldots, n)$. De plus, en utilisant la remarque qui suit, les théorèmes 1, 2 et un récent résultat de F. Gantmacher et M. Krein qui va être publié, on peut établir le

Théorème 3. *Soient*

$$\omega_1(x; \lambda), \; \omega_2(x; \lambda), \ldots, \omega_q(x; \lambda) \; (-\infty < \lambda < \infty)$$

quelques q $(1 < q < n; \; p+q=n)$ *solutions linéairement indépendantes du système différentiel*

$$\left.\begin{aligned} & D^n y - \lambda \rho y = 0 \\ & \sum_{k=0}^{n-1} A_{ik} D^k y \Big|_{x=a} = 0 \; (i=1, 2, \ldots, p) \end{aligned}\right\}$$

où la matrice

$$\left\| \begin{matrix} A_{10} A_{11} \ldots \ldots \ldots A_{1n-1} \\ \ldots \ldots \ldots \ldots \ldots \\ A_{p0} A_{p1} \ldots \ldots \ldots A_{pn-1} \end{matrix} \right\|$$

possède la propriété, que tous ses mineurs d'ordre p *sont non-négatifs (et ne sont pas tous* $=0$*).*

Alors la fonction $W(x; \lambda) = W(\omega_1, \ldots, \omega_q)$ *admet le développement suivant*

$$W(x; \lambda) = W(x; 0) \prod_{k=0}^{\infty} \left(1 - \frac{\lambda}{\lambda_k(x)}\right)$$

où

a) $W(x; 0) \neq 0$ *pour* $a < x \leqslant b$.

b) $0 < (-1)^q \lambda_0(x) < (-1)^q \lambda_1(x) < (-1)^q \lambda_2(x) < \ldots$

c) *la fonction* $(-1)^k \lambda_k(x)$ $(k=1, 2, \ldots)$ *croît de* $\lambda_k = \lambda_k(b)$ *à* ∞ *quand* x *décroît de* b *à* a $(a < x \leqslant b)$.

Par conséquent, la fonction $W(x; \lambda)$ possède aussi les propriétés 1°, 2°, 3° indiquées dans le théorème 1.

De même, si l'on définit maintenant la fonction $\varphi_q(x; \lambda)$ comme une solution du système différentiel

$$D^n y - \lambda \rho y = 0, \sum_{k=0}^{n-1} A_{ik} D^k y \Big|_{x=a} = 0 \quad (i=1, 2, \ldots, p)$$

$$D^i y \Big|_{x=b} = 0 \quad (i=0, 1, \ldots, q-2)$$

elle conserve les propriétés 1°, 2° indiquées dans le théorème 2.

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MATHÉMATIQUES

SUR LA DÉCOMPOSITION DES FONCTIONNELLES EN COMPOSANTES POSITIVES

Par J. GROSBERG et M. KREIN

(Présenté par A. N. Kolmogoroff, de l'Académie, le 27. X. 1939)

Soit E un espace linéaire normé, et $K (K \subset E)$ un ensemble possédant les propriétes suivantes:

1°. *Si* $x \in K$ *et* $y \in K$, *on a* $x+y \in K$.

2°. *Si* $x \in K$ *et* $\lambda \geqslant 0$, *on a* $\lambda x \in K$.

3°. *Si* $x \in K$ *et* $x \neq 0$, *on a* $-x \overline{\in} K$.

L'ensemble K étant donné, on peut considérer l'espace E comme un espace semi-ordonné dans un certain sens, en entendant la relation $x < y$ $(x, y \in E)$ comme équivalente à $y - x \in K$ et $y \neq x$.

Cela posé, l'espace conjugué E^* devient aussi un espace semi-ordonné, si l'on convient d'écrire $g < f (g, f \in E^*)$ lorsque $g \neq f$ et

$$g(x) \leqslant f(x) \quad \text{pour } x > 0.$$

C étant une constante quelconque $\geqslant 1$, convenons de dire que l'ensemble K satisfait à la condition C, s'il possède la propriété suivante:

4°. *Si* $y < x < z$, $|y| \leqslant 1$, $|z| \leqslant 1$, *on a* $|x| \leqslant C$.

Disons encore qu'une fonctionnelle linéaire $f (f \in E^*)$ admet une décomposition C, si elle admet la représentation suivante:

$$f = g - h \tag{1}$$

où les fonctionnelles linéaires $g, h \in E^*$ sont positives ou nulles, $g, h \geqslant 0$, et sont liées entre elles par la relation

$$|g| + |h| \leqslant C |f|. \tag{2}$$

Si $C = 1$, le signe $<$ est exclu et l'on obtient

$$|g| + |h| = |f|.$$

Dans ce cas la décomposition (1) sera dite *canonique*.

En supposant que l'ensemble K possède les propriétés 1°, 2°, 3° et contient d'ailleurs des points intérieurs, M. Krein a trouvé des conditions nécessaires et suffisantes pour qu'il existe un C suffisamment grand, pour lequel toute fonctionnelle $f \in E^*$ admet une décomposition C [voir «Sur quelques questions de la théorie des moments» de Achyeser et Krein, art. III, p. 166, th. 8 (1)]. En particulier, M. Krein a établi

le théorème suivant (voir la note du théorème 4 de l'article cité ci-dessus).

A. *S'il existe dans K un élément u de norme $=1$ ($|u|=1$) et tel que l'ensemble de tous les éléments $x \in E$ satisfaisant aux inégalités*

$$-u < x < u$$

contient la sphère ouverte $|x|<1$, chaque fonctionnelle $f \in E^*$ admet une décomposition canonique.

En partant de cette proposition nous allons démontrer le théorème général suivant:

Théorème 1. *L'ensemble $K \subset E$ possédant les propriétés 1°, 2°, 3°, la condition nécessaire et suffisante pour que toute fonctionnelle $f \in E^*$ admette une décomposition C est que l'ensemble K satisfasse à la condition C.*

Démonstration. La condition est nécessaire. En effet, soit

$$y < x < z \quad (|y| \leqslant 1,\ |z| \leqslant 1). \tag{3}$$

Il existe une fonctionnelle $f \in E^*$ telle que

$$f(x) = x, \quad |f| = 1.$$

Soit

$$f = g - h \quad (g, h \geqslant 0) \tag{4}$$

la décomposition de f, donc $|g| + |h| \leqslant C$.

Comme $g, h \geqslant 0$, il suit de (3) que

$$-|g| \leqslant g(y) \leqslant g(x) \leqslant g(z) \leqslant |g|$$
$$-|h| \leqslant h(y) \leqslant h(x) \leqslant h(z) \leqslant |h|.$$

D'où en vertu de (4) on a

$$|x| = f(x) = g(x) - h(x) \leqslant |g| + |h| \leqslant C.$$

La condition est suffisante. Désignons par E_1 le produit abstrait de l'espace E par l'axe réel (c'est-à-dire, l'ensemble de toutes les paires (x, t), $x \in E$, $-\infty < t < \infty$).

Si nous convenons d'écrire x au lieu de $(x, 0)$ et w au lieu de $(0, 1)$, on pourra mettre tout élément $(x, t) \in E_1$ sous la forme $x + tw$.

Formons dans l'espace E_1 l'ensemble K_1 (possédant les propriétés 1°, 2°, 3°), en le définissant par la condition

$$x + tw \in K_1$$

si $t \geqslant 0$ et s'il existe un élément $y \in E$ tel que

$$x + ty \geqslant 0, \quad |y| \leqslant 1.$$

Il est évident, que l'intersection de K_1 avec E est l'ensemble K, donc le sens des relations d'ordre pour les éléments E reste le même soit que l'on part de K ou bien de K_1.

Introduisons dans l'espace E_1 la norme $|z|_c$ ($z \in E_1$) en posant

$$|z|_c = \inf a, \tag{5}$$

où a est un nombre positif, satisfaisant à l'inégalité

$$-aw \leqslant z \leqslant aw.$$

Il est aisé de vérifier que la quantité $|z|_c$ ainsi définie possède toutes les propriétés nécessaires de la norme.

On obtient ainsi pour les élements de E deux normes $|x|$ et $|x|_c$ (la norme canonique). Montrons que ces deux normes sont topologiquement équivalentes dans E. En effet, en mettant x sous la forme $x=|x|e$ ($|e|=1$) on a $\pm x+|x|(\mp e)=0$, donc, d'après la définition de K_1, on peut écrire

$$\pm x+|x|w\in K, \quad \text{ou} \quad -w|x|<x<w|x|.$$

D'où d'après la définition (5) $|x|_c\leqslant|x|$. D'autre part, on a pour tout $\varepsilon>0$

$$-w(|x|_c+\varepsilon)<x<w(|x|_c+\varepsilon).$$

D'après la définition de K_1, cette inégalité signifie qu'il existe des $y, z\in E$ tels que

$$z(|x|_c+\varepsilon)<x<y(|x|_c+\varepsilon), \quad |z|\leqslant 1, \quad |y|\leqslant 1.$$

D'où, il résulte en vertu de la condition 4° que

$$|x|\leqslant C(|x|_c+\varepsilon).$$

Ainsi on trouve définitivement que

$$|x|_c\leqslant|x|\leqslant C|x|. \tag{6}$$

En remplaçant dans E la norme $|x|$ par la norme $|x|_c$, on ne change pas évidemment l'ensemble E^* de toutes les fonctionnelles linéaires. Pourtant une telle opération change la norme $|f|$ de la fonctionnelle $f\in E^*$. En désignant par $|f|_c$ la norme correspondant à la norme $|x|_c$, il est facile de conclure qu'en vertu de (6) on aura

$$|f|\leqslant|f|_c\leqslant C|f|. \tag{7}$$

Observons maintenant que l'ensemble K_1 satisfait à la condition du théorème A (pour $u=w$). En effet, $|w|_c=1$ et en outre si

$$|z|_c<1 \quad (z\in E_1)$$

on a par définition

$$-w<z<w.$$

Cela étant, toute fonctionnelle $F\in E_1^*$ admet la décomposition

$$F=G-H \tag{8}$$

où

$$G\geqslant 0, \qquad H\geqslant 0.$$

Soit maintenant $f\neq 0$ une fonctionnelle arbitraire de E^*. Formons une fonctionnelle $F(x)$ qui présente le prolongement de f sur E_1 et telle que

$$|f|_c=|F|.$$

Considerons sa décomposition (8). Définissons les fonctionnelles $g, h\in E^*$ par les égalités

$$g(x)=G(x), \quad h(x)=H(x) \quad (x\in E).$$

Il est évident que

$$f=g-h, \quad g\geqslant 0, \quad h\geqslant 0 \tag{9}$$

et qu'en outre

$$|g|_c+|h|_c\leqslant|G|+|H|=|F|=|f|_c.$$

Par conséquent, en vertu de (7) on a

$$|g|+|h|\leqslant|g|_c+|h|_c=|f|_c\leqslant C|f|$$

donc, la décomposition (9) est la décomposition C cherchée de la fonctionnelle f.

Le théorème est donc démontré.

2. Soit E' un espace linéaire normé (généralement différent de E) qui est semi-ordonné au moyen d'un ensemble conique K'. Convenons de dire que E et E' sont *semblablement isomorphes*, si l'on peut établir entre E et E' une isomorphie * $x \longleftrightarrow x'$ $(x \in E,\ x' \in E')$ telle que l'inégalité $x < y$ $(x \in E,\ y \in E)$ implique l'inégalité $x' < y'$ $(x \longleftrightarrow x',\ y \longleftrightarrow y')$ et inversement.

Si E' est une partie linéaire de l'espace $C(0, 1)$ de toutes les fonctions continues $x(t)$ $(0 \leqslant t \leqslant 1)$ (avec la définition de la norme $\|x\| = \max|x(t)|$), il est naturel d'entendre sous $K' \subset E'$ l'ensemble de toutes les fonctions non-négatives.

En supposant que l'ensemble $K \subset E$ contient des points intérieurs, M. Krein a trouvé dans un travail qui va être publié, des conditions nécessaires et suffisantes pour que l'espace E soit semblablement isomorphe à un sous-espace de $C(0,1)$. En particulier, il a établi la proposition suivante.

B. *Si l'espace E est séparable et l'ensemble K étant fermé satisfait aux conditions du théorème* A, *l'espace E est semblablement isomorphe à un sous-espace de l'espace des fonctions continues C (0,1) (d'ailleurs, l'isomorphie dont il s'agit peut être choisie de manière qu'elle soit aussi isométrique).*

Il suit de cette proposition et de nos considérations précédentes que l'on a le

Théorème 2. *Soit $K \subset E$ un ensemble fermé possédant les propriétés* 1°, 2°, 3°. *Pour que l'espace E soit semblablement isomorphe à un sous-espace de l'espace des fonctions continues C (0,1), il faut et il suffit qu'il soit séparable et que pour un $C \geqslant 1$ l'ensemble K vérifie la condition C.*

Démonstration. La condition est nécessaire, car si E est semblablement isomorphe à un sous-espace $E \subset C$ (0,1), E étant séparable, l'espace E l'est aussi. D'ailleurs, comme la condition C est évidemment vérifiée dans E' pour $C = 1$, elle est aussi vraie dans E pour un C suffisamment grand.

La condition est suffisante, car si elle est vérifiée, l'espace E est semblablement isomorphe à l'espace E_1 qu'on a construit en démontrant le théorème 1. D'autre part, d'après la proposition B l'espace E_1 est semblablement isomorphe à un sous-espace de C (0,1).

Le théorème est donc démontré.

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* C'est-à-dire une correspondance biunivoque, linéaire et continue entre les éléments de E et de E'.

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MATHÉMATIQUES

LES FONCTIONS PRESQUE ABSOLUMENT MONOTONES

Par I. CHLODOVSKY

(Présenté par S. N. Bernstein, de l'Académie, le 24. X. 1939)

1. M. Serge Bernstein (1) a étudié les propriétés essentielles des fonctions absolument monotones. Dans cette étude M. Bernstein partait de la définition classique d'une fonction absolument monotone, c'est-à-dire d'une fonction dont toutes les différences finies sont non-négatives. Cette définition peut être remplacée par une définition équivalente qui admet une généralisation bien naturelle et qui permet d'abréger la démonstration de plusieurs théorèmes de la théorie des fonctions absolument monotones.

Les résultats de M. Hausdorff (2) concernant la théorie des moments. permettent de définir une fonction absolument monotone de la manière suivante:

Définition 1. *Une fonction $F(x)$ est dite absolument monotone dans un segment $[a, b]$, si chaque suite arbitraire de valeurs*

$$F(x_0), \quad F(x_1), \quad \ldots, \quad F(x_n) \tag{1}$$

où $b \geqslant x_0$, $x_n \geqslant a$, $x_k - x_{k+1} = \delta > 0$ est la suite des moments d'une fonction $\omega(x)$ non-décroissante dans le segment $[0, 1]$, c'est-à-dire si

$$F(x_k) = \int_0^1 x^k d\omega(x) \qquad (k = 0, 1, 2, \ldots, n).$$

Cette définition se généralise naturellement ainsi:

Définition 2. *Une fonction $F(x)$ est dite presque absolument monotone dans un segment $[a, b]$, si chaque suite arbitraire de ses valeurs* (1) *est la suite des moments d'une fonction non-décroissante définie dans un segment $[0, l(\delta)]$ (le cas $l(\delta) = \infty$ est aussi possible).*

En particulier, $F(x)$ sera presque absolument monotone sur la demi-droite $-\infty < x \leqslant 0$, si la suite des valeurs $\{F(\alpha + \kappa\delta)\}$, $\alpha \leqslant 0$, $\delta < 0$, est la suite des moments d'une fonction non-décroissante dans un segment $[0, l(\delta)]$.

2. Les méthodes classiques de Stieltjes (3) et les méthodes de l'analyse fonctionnelle dont F. Riesz (4) est le fondateur permettent de démontrer les propriétés des fonctions presque absolument monotones que nous indiquons dans la suite. Dans le présent article nous nous bornons au cas des fonctions presque absolument monotones définies sur la demi-droite $-\infty < x \leqslant 0$.

Remarquons d'abord que pour la classe des fonctions que nous étudions on peut donner une définition équivalente à la définition 2 et qui est une généralisation de la définition classique.

Définition 3. *La fonction $F(x)$ est dite presque absolument monotone dans $-\infty < x \leqslant 0$, si quels que soient les nombres négatifs α et δ et l'entier N, il existe un nombre positif β qui dépend, en général, de α, δ et N et tel que toutes les différences finies généralisées*

$$\Delta_\beta^n F(\alpha) = F(\alpha) - \binom{n}{1}\frac{1}{\beta}F(\alpha+\delta) + \binom{n}{2}\frac{1}{\beta^2}F(\alpha+2\delta) - \ \ldots\ + \pm\frac{(-1)^n}{\beta^n}F(\alpha+n\delta)$$

où $n = N-k$ et $k = 0, 1, \ldots N$, sont non-négatives.

Remarquons que la définition 3 peut être facilement énoncée sous forme d'un critère *nécessaire et suffisant* pour qu'une suite de nombres $\{c_k\}$ soit la suite des moments d'une fonction non-décroissante dans $0 \leqslant x < \infty$.

3. Désignons par $\Phi(x)$ une fonction presque absolument monotone dans $-\infty < x \leqslant 0$. On a pour $\Phi(x)$ les théorèmes suivants.

1°. *$\Phi(x)$ est une fonction analytique dans le demi-plan $R(x) < 0$. $\Phi(x)$ peut être présentée d'une manière unique dans la forme*

$$\Phi(x) = \int_{-\infty}^{\infty} e^{xt}\, d\omega(t)$$

où $\omega(t)$ est une fonction non-décroissante normée par les conditions

$$\omega(-\infty) = 0, \quad \omega(x) = \frac{1}{2}[\omega(x-0) + \omega(x+0)].$$

2°. *Si une certaine suite de valeurs $\{\Phi(\alpha+k\delta)\}$ est la suite des moments d'une fonction non-décroissante dans un segment fini $[0, l(\alpha, \delta)]$, alors pour un $x < 0$ fixé quelconque et pour un δ arbitraire négatif la suite $\{\Phi(x+k\delta)\}$ est une suite de moments dans un intervalle fini.*

3°. Nous allons nommer ordre de $\Phi(x)$ la longueur du plus petit intervalle $[0, \beta]$, à l'intérieur duquel la suite $\{\Phi(-k)\}$ est la suite des moments d'une fonction non-décroissante. *Pour que $\Phi(x)$ soit d'ordre β fini, il faut et il suffit que*

$$\lim_{n\to\infty} \frac{\Phi(nx)}{\Phi(n+1x)} = \beta^{-x}.$$

Si $\Phi(x)$ est d'ordre β, l'on a

$$\Phi(x) = \beta^{-x} f(x)$$

où $f(x)$ est absolument monotone pour $-\infty < x \leqslant 0$.

4°. *Tous les déterminants de Hankel $|\Phi(k\delta)|_{k=0}^{n}$ et $|\Phi(k\delta)|_{k=1}^{n}$ sont non-négatifs. Si le premier déterminant qui ne s'annulle pas est de la forme $|\Phi(k\delta)|_{k=0}^{n}$, on a*

$$\Phi(x) = \alpha_1\lambda_1^{-x} + \alpha_2\lambda_2^{-x} + \ \ldots\ + \alpha_n\lambda_n^{-x}, \quad \text{où} \quad \alpha_k > 0 \quad \text{et} \quad \lambda_k > 0.$$

Si le premier déterminant qui ne s'annulle pas est de la forme $|\Phi(k\delta)|_{k=1}^{n}$, on a

$$\Phi(x) = \int_0^{\beta} t^{-x}\, dS(t)$$

où $S(t)$ est une fonction discontinue en $x = 0$ ayant un nombre fini de points de discontinuité et constante entre ces points.

5°. *Pour que* $\Phi(x)$ *puisse être définie par un nombre fini de ses valeurs, il est nécessaire et suffisant que l'on ait*

$$|\Phi(k\delta)|_0^n > 0 \qquad 2n-1 \leqslant n < N \qquad |\Phi(k\delta)|_1^n > 0 \qquad 2n \leqslant m < N$$
$$|\Phi(k\delta)|_0^n = 0 \qquad m < 2n-1 \leqslant N \qquad |\Phi(k\delta)|_1^n = 0 \qquad m < 2n \leqslant N$$

Ainsi, par exemple, 2^{-x} est la seule fonction presque absolument monotone dans $(-\infty, 0)$, qui est définie par les deux valeurs $\Phi(0)=1$ et $\Phi(-1)=2$.

6°. *Toutes les fonctions* $\Phi(x)$ *presque absolument monotones qui possèdent aux points* $x=k\delta$ *des valeurs données* $\{\Phi(k\delta)\}$ *sont bornées par deux fonctions* $f_\delta(x)$ *et* $\varphi_\delta(x)$ *presque absolument monotones de la manière suivante:*

$$\Phi(k\delta) = f_\delta(k\delta) = \varphi_\delta(k\delta), \qquad k=0,\ 1,\ 2,\ \ldots$$
$$f_\delta(x) \leqslant \Phi(x) \leqslant \varphi_\delta(x), \quad \textit{si} \quad 2k\delta < x < \overline{2k+1}\,\delta$$
$$\varphi_\delta(x) \leqslant \Phi(x) \leqslant f_\delta(x), \quad \textit{si} \quad \overline{2k+1}\,\delta < x < 2k\delta.$$

7°. *Si* $\Phi(x)$ *est continue à gauche au point* $x=0$, *la suite des valeurs* $\{\Phi(k\delta)\}$ *détermine* $\Phi(x)$ *d'une manière unique dans le cas et dans ce cas seulement où*

$$\rho(\delta) = \lim_{n\to\infty} \frac{|\Phi(k\delta)|_{k=1}^{n}}{|\Phi(k\delta)|_{k=2}^{n+1}} = 0.$$

Si $\Phi(x)$ *est discontinue à gauche au point* $x=0$, *pour que la suite de ses valeurs* $\{\Phi(k\delta)\}$ *détermine* $\Phi(x)$ *d'une manière unique il faut et il suffit que l'on ait*

$$r(\delta) = \lim_{n\to\infty} \frac{|\Phi(k\delta)|_{k=1}^{n}}{\begin{vmatrix} 0 & \Phi(0) & \Phi(\delta) & \ldots & \Phi(\overline{n-1}\,\delta) \\ \Phi(0) & \Phi(\delta) & \Phi(2\delta) & \ldots & \Phi(n\delta) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \Phi(\overline{n-1}\,\delta) & \Phi(n\delta) & \ldots & \Phi(\overline{2n-1}\,\delta) \end{vmatrix}} = 0.$$

8°. Un nombre réel δ sera nommé *nombre d'interpolation* de la fonction $\Phi(x)$, si la suite des valeurs $\{\Phi(k\delta)\}$ détermine $\Phi(x)$ d'une manière unique.

Si δ *est un nombre d'interpolation de* $\Phi(x)$, *un nombre arbitraire négatif* $\delta^1 > \delta$, $(|\delta^1| < |\delta|)$, *sera aussi un nombre d'interpolation de* $\Phi(x)$.

9°. *La suite des nombres* $\{\Phi^{(k)}(\alpha)\}$ *où* $\alpha < 0$ *sera la suite des moments d'une fonction non-décroissante dans* $-\infty < x < \infty$, *si l'ordre de* $\Phi(x)$ *est* > 1, *et la suite des moments d'une fonction non-décroissante pour* $0 \leqslant x < \infty$, *si l'ordre de* $\Phi(x)$ *est* $\leqslant 1$.

Remarquons que le théorème 9 permet d'établir un principe de dualité, suivant lequel il correspond à chaque théorème concernant les valeurs de $\{\Phi(k\delta)\}$ un théorème concernant la suite des valeurs des dérivées $\{f_{(0)}^{(k)}\}$ d'une fonction absolument monotone.

En particulier, on peut obtenir ainsi tous les théorèmes de M. Bernstein exposés dans le premier chapitre du Mémoire précédemment cité. Indiquons l'exemple suivant de l'application de la loi de dualité qui conduit à un résultat qui n'est pas cité explicitement dans le Mémoire de M. Bernstein.

Nous dirons que la fonction presque absolument monotone

$$\Phi(x) = \int_0^\infty t^{-x}\, d\omega(t), \qquad x \leqslant 0$$

est conjuguée à la fonction absolument monotone

$$f(x)=\int_0^\infty e^{xt}\,d\omega(t), \qquad x\leqslant 0.$$

Une fonction $f(x)$ absolument monotone pour $x\leqslant 0$ a une fonction conjuguée dans le cas et dans ce cas seulement, où elle possède toutes les dérivées pour $x=0$. Il est facile à voir que

$$f_{(0)}^{(k)}=\Phi(-k), \qquad k=0,\ 1,\ 2,\ \ldots$$

L'inversion du théorème 7° conduit au théorème

10°. *Pour qu'une fonction $f(x)$ absolument monotone pour $x\leqslant 0$ soit définie d'une manière unique par la suite*

$$f_{(0)},\quad f_{(0)}^{(l)},\quad f_{(0)}^{(2l)},\ \ldots\ f_{(0)}^{(nl)},\ \ldots$$

où l est un entier positif arbitraire, il faut et il suffit que

$$\lim_{n\to\infty}\frac{|f_{(0)}^{(kl)}|_{k=1}^{n}}{|f_{(0)}^{(kl)}|_{k=2}^{n}}=0,$$

si la fonction conjuguée $\Phi(x)$ est continue en $x=-0$, ou bien que

$$\lim_{n\to\infty}\frac{|f_{(0)}^{(kl)}|_{k=1}^{n}}{\begin{vmatrix} 0 & f_{(0)}^{(l)} & f_{(0)}^{(2l)} & \ldots & f_{(0)}^{(n-1l)} \\ f_{(0)}^{(l)} & f_{(0)}^{(2l)} & \ldots & f_{(0)}^{(nl)} \\ \ldots & \ldots & \ldots & \ldots \\ f_{(0)}^{(n-1l)} & f_{(0)}^{(nl)} & \ldots & f_{(0)}^{(2n-1l)} \end{vmatrix}}=0$$

si $\Phi(x)$ est discontinue au point $x=-0$.

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MATHÉMATIQUES

SUR LES SOLUTIONS PÉRIODIQUES DE CERTAINES ÉQUATIONS NON-LINÉAIRES DU QUATRIÈME ORDRE

Par P. V. SOLOVIEFF

(*Présenté par S. L. Soboleff, de l'Académie, le 16. X. 1939*)

Considérons l'équation différentielle suivante:

$$\frac{\partial^2 z}{\partial t^2}+\frac{(2k+1)^2}{\pi^2}\frac{\partial^4 z}{\partial x^4}=\Phi(x,t)+\mu f(z) \tag{1}$$

(où k est un nombre entier quelconque) dans la région $\overline{D}=D\left\{\begin{matrix}0\leqslant x\leqslant 1\\ 0\leqslant t\leqslant 1\end{matrix}\right\}$.

Supposons que la fonction $\Phi(x,t)$ satisfait à la condition de Lipschitz

$$|\Phi(x_1,t_1)-\Phi(x_2,t_2)|\leqslant A|x_1-x_2|+B|t_1-t_2|$$

dans la région $\overline{D}$ et que les fonctions

$$\Phi_1(x,t)=\frac{\partial\Phi}{\partial x};\ \Phi_2(x,t)=\frac{\partial\Phi}{\partial t}$$

sont des fonctions continues à variation bornée par rapport au premier argument pour chaque $0\leqslant t\leqslant 1$, et sont des fonctions à variation bornée par rapport au second argument pour chaque $0\leqslant x\leqslant 1$ *. Supposons d'ailleurs que la fonction $\Phi(x,t)$ est développable en série de Fourier

$$\Phi(x,t+1)=\Phi(x,t)=\sum_{n=0}^{\infty}\Phi_{2n+1}(t)\sin(2n+1)\pi x \tag{2}$$

où

$$\Phi_{2n+1}(t)=\Phi_{2n+1}(t+1)=2\int_0^1\Phi(\sigma,t)\sin(2n+1)\pi\sigma\,d\sigma.$$

Nous allons nommer conditions (A) les conditions auxquelles la fonction $\Phi(x,t)$ doit satisfaire.

Cherchons une solution de l'équation (1), satisfaisant aux conditions:

$$\left.\begin{aligned} z(0,t)&=\frac{\partial^2 z}{\partial x^2}\Big|_{x=0}=0,\quad z(x,0)=z(x,1)\\ z(1,t)&=\frac{\partial^2 z}{\partial x^2}\Big|_{x=1}=0,\quad \frac{\partial z}{\partial t}\Big|_{t=0}=\frac{\partial z}{\partial t}\Big|_{t=1}\end{aligned}\right\} \tag{3}$$

* Pour abréger, nous allons nommer dans la suite les fonctions de cette nature les fonctions à variation bornée par rapport au premier et second argument.

Théorème I. *Une équation différentielle* (1) *étant donnée, nous allons obtenir dans la région* $\overline{D}$ *une solution continue, ainsi que ses dérivées partielles du quatrième ordre* *, *satisfaisant aux conditions* (3) *et périodique par rapport à l'argument t, de période un, si:*

1) *La fonction* $\Phi(x,t)$ *satisfait aux conditions* (A).

2) $|f'(z_1)-f'(z_2)\leqslant M|z_1-z_2|$; $f(-z)=-f(z)$; $f(0)=0$.

3)** $|\mu|<\frac{|a|\pi^2}{AN}$, *où* $a=\frac{2k+1}{\pi}$ (k *est un nombre entier quelconque*).

$N=\sup|f'(z)|$, *pour les valeurs finies de* z;

$$A=\sup\left[2\int_0^1 d\sigma\int_0^1\left|\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi x\sin(2n+1)\pi\xi\sin(2n+1)^2\pi^2 a(t-\sigma)}{(2n+1)^2}\right|d\xi+\right.$$

$$\left.+\int_0^1 d\sigma\int_0^1\left|\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi x\sin(2n+1)\pi\xi\cos(2n+1)^2\pi^2 a\left(t-\sigma+\frac{1}{2}\right)}{(2n+1)^2\sin\frac{(2n+1)^2\pi^2 a}{2}}\right|d\xi\right]=$$

$$=\sup\left[\int_0^t d\sigma\int_0^1|K_1(x,t;\ \xi,\sigma)|\,d\xi+\int_0^1 d\sigma\int_0^1|K_2(x,t;\ \xi,\sigma)|\,d\xi\right].$$

Pour démontrer ce théorème on peut appliquer la méthode des approximations successives.

Prenons comme approximation initiale $z_0(x,t)$ la solution de l'équation:

$$\frac{\partial^2 z_0}{dt^2}+a^2\frac{\partial^4 z}{\partial x^4}=\Phi(x,t) \qquad (4)$$

satisfaisant aux conditions (3). Cette solution aura la forme:

$$z_0(x,t)=\sum_{n=0}^{\infty}S^0_{2n+1}(t)\sin(2n+1)\pi x \qquad (5)$$

où

$$S^0_{2n+1}(t)=\frac{2}{a\pi^2(2n+1)^2}\int_0^t d\sigma\int_0^1\Phi(\xi,\sigma)\sin(2n+1)\pi\xi\sin(2n+1)^2\pi^2 a(t-\sigma)\,d\xi+$$

$$+\frac{1}{a\pi^2(2n+1)^2\sin\frac{(2n+1)^2\pi^2 a}{2}}\int_0^1 d\sigma\int_0^1\Phi(\xi,\sigma)\sin(2n+1)\pi\xi\cos(2n+1)^2\times$$

$$\times\pi^2 a\left(t-\sigma+\frac{1}{2}\right)d\xi.$$

En vertu des conditions, qu'on a imposé à la fonction $\Phi(x,t)$, la série (5) et toutes les séries que l'on obtient en la différentiant, terme à terme, deux fois par t et quatre fois par x, seront absolument et uniformément convergentes dans la région $\overline{D}$.

Prenons, comme première approximation $z_1(x,t)$ la solution de l'équation:

$$\frac{\partial^2 z_1}{\partial t^2}+a^2\frac{\partial^4 z_1}{\partial x^4}=\Phi(x,t)+\mu f[z_0] \qquad (6)$$

---

* Savoir: la seconde dérivée par rapport à t et la quatrième par rapport à x.

** Le nombre a est la racine carrée du coefficient de la quatrième dérivée par rapport à x de l'équation (1).

dans les mêmes conditions (3), etc. La k-me approximation $z_k(x, t)$ aura la forme:

$$z_k(x, t) = \sum_{n=0}^{\infty} S_{2n+1}^k(t) \sin(2n+1)\pi x \qquad (7)$$

où

$$S_{2n+1}^k(t) = \frac{2}{a\pi^2(2n+1)^2} \int_0^t d\sigma \int_0^1 [\Phi(\xi, \sigma) + \mu f(z_{k-1})] \sin(2n+1)\pi\xi \times$$

$$\times \sin(2n+1)^2 \pi^2 a(t-\sigma)\, d\xi + \frac{1}{a\pi^2(2n+1)^2 \sin\frac{(2n+1)^2\pi^2 a}{2}} \int_0^1 d\sigma \int_0^1 [\Phi(\xi, \sigma) +$$

$$+ \mu f(z_{k-1})] \sin(2n+1)\pi\xi \cos(2n+1)^2 \pi^2 a \left(t - \sigma + \frac{1}{2}\right) d\xi.$$

Alors la série:

$$z(x, t) = z_0 + [z_1 - z_0] + \ldots + [z_k - z_{k-1}] + \ldots \qquad (8)$$

est absolument et uniformément convergente dans la région $\overline{D}$, parce que la série majorante

$$I = L \sum_{n=0}^{\infty} \left[\left| \frac{\mu}{a} \right| \frac{NA}{\pi^2} \right]^n \quad (L \geqslant |z_0|)$$

est convergente en vertu de la condition (3) du théorème I. En changeant l'ordre de la sommation et de l'intégration de la série (8) nous obtenons:

$$z(x, t) = \frac{2}{a\pi^2} \int_0^t d\sigma \int_0^1 [\Phi(\xi, \sigma) +$$

$$+ \mu f(z)] \sum_{n=0}^{\infty} \frac{\sin(2n+1)\pi x \sin(2n+1)\pi\xi \sin(2n+1)^2\pi^2 a(t-\sigma)}{(2n+1)^2}\, d\xi +$$

$$+ \frac{1}{a\pi^2} \int_0^t d\sigma \int_0^1 [\Phi(\xi, \sigma) +$$

$$+ \mu f(z)] \sum_{n=0}^{\infty} \frac{\sin(2n+1)\pi x \sin(2n+1)\pi\xi \cos(2n+1)^2\pi^2 a\left(t-\sigma+\frac{1}{2}\right)}{(2n+1)^2 \sin\frac{(2n+1)^2\pi^2 a}{2}}\, d\xi. \qquad (9)$$

De ce qui précède on voit aisément que la fonction $z(x, t)$ est une fonction à variation bornée, par rapport au premier et au second argument. Cette remarque étant faite, on peut démontrer facilement la convergence absolue et uniforme de la série:

$$\frac{\partial z}{\partial x} = \frac{\partial z_0}{\partial x} + \left[\frac{\partial z_1}{\partial x} - \frac{\partial z_0}{\partial x}\right] + \ldots + \left[\frac{\partial z_k}{\partial x} - \frac{\partial z_{k-1}}{\partial x}\right] + \ldots \qquad (10)$$

dans la région $\overline{D}$. En différentiant la série (8) deux fois par rapport à t et ensuite quatre fois par rapport à x, nous allons obtenir des séries absolument et uniformément convergentes. La fonction $z(x, t)$, représentée par la formule (9), sera une solution périodique de l'équation (1).

Théorème II. *L'équation différentielle* (1) *dans les conditions* (3) *n'a qu'une seule solution périodique dans la classe des fonctions développables en séries de Fourier de sinus* *, *si les conditions du théorème* I *sont vérifiées.*

* La série de Fourier de la forme (2).

Supposons qu'il existe deux solutions différentes de l'équation (1) continues par rapport à z et u et satisfaiant aux conditions (3). Alors $\psi = u - z$ sera la solution de l'équation:

$$\frac{\partial^2 \psi}{\partial t^2} + a^2 \frac{\partial^4 \psi}{\partial x^4} = \mu [f(u) - f(z)].$$

Par conséquent, elle est aussi une solution de l'équation

$$\psi = u - z = \frac{\mu}{a\pi^2} \int_0^t d\sigma \int_0^1 [f(u) - f(z)] K_1(x, t; \xi, \sigma) d\xi +$$

$$+ \frac{\mu}{a\pi^2} \int_0^1 d\sigma \int_0^1 [f(u) - f(z)] K_2(x, t; \xi, \sigma) d\xi$$

d'où

$$\sup |u - z| \leqslant \left| \frac{\mu}{a} \right| NA \sup |u - z|$$

ou

$$|\mu| \geqslant \frac{\pi^2 |a|}{AN},$$

ce que contredit à la condition du théorème.

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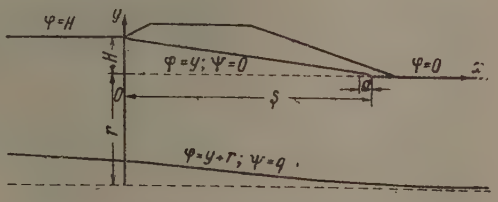
HYDRAULIK

GRUNDWASSERBEWEGUNG IM KÖRPER UND IN DER UNTERLAGE EINES HOMOGENEN ERDDAMMES MIT HORIZONTALEM FILTER BEI ENDLICHER TIEFE DER DURCHLÄSSIGEN UNTERGRUNDSCHICHT (EBENES PROBLEM)

Von A. VOSTSCHININ

(*Vorgelegt von N. E. Kotchin, Mitglied der Akademie, d. 5. X. 1939*)

Die Theorie der Grundwasserbewegung von Akademiker Pawlowsky bietet die Möglichkeit einer Anwendung hydrodynamischer Methoden zur Lösung praktisch wichtiger und alltäglicher Aufgaben des Wasserbaues. Hinsichtlich der Lösungsmethode unterscheidet sich die vorliegende Aufgabe nicht von einer ganzen Reihe anderer, seit 1922 im Schrifttum entstandenen Probleme, in denen verschiedene, praktisch notwendige Formen des Bereiches der Grundwasserbewegung behandelt werden.



Beim Anlegen des Kuibyshew Erddammes an der Wolga stellte es sich als notwendig eine neue Form des Grundwasserstrombereiches zu untersuchen. Übrigens ist diese Form allen homogenen Erddämmen eigen, deren Untergrund ein durchlässiges auf undurchlässiger Sohle ruhendes Boden darstellt. Die Aufgabe ist dadurch von einer gewissen allgemeinen Bedeutung.

Die Form der zu behandelnden Grundstrombewegung ist aus dem folgenden Schema zu ersehen.

Bezeichnungen: H — wirkendes Gefälle; T — Tiefe des untersten Punktes der wasserdichten Sohle; S — Länge des Dammes von der Oberwasserlinie am Dammkörper bis zum Austrittspunkt der Sickerungskurve; a — Anfangspunkt der ins Innere der Strömung eingreifender Dräne; φ — Potentialfunktion; ψ — Funktion der Strömung.

Gemäss dem zu behandelnden Schema baut man auf den entsprechenden komplexen Ebenen die Änderungsbereiche zweier Funktionen der komplexen Veränderlichen, deren eine

$$w = \varphi(x, y) + i\psi(x, y) \qquad (1)$$

als «komplexer Potential» bezeichnet wird und die andere

$$\Omega = \Phi + iF = \dot{w} + iz \qquad (2)$$

die Benennung «Joukowsky's Komplex» erhalten hat, wo $z=x+iy$ die Koordinate eines Punktes des Bereiches der Grundstrombewegung bedeutet. Der erste Bereich ergibt sich als ein Rechteck der Breite H und der Länge q, und der zweite — als ein unendlicher Streifen der Breite T.

Nach dem konformen Abbilden dieser Bereiche auf eine und dieselbe Halbebene, gemäss der Formel von Schwarz-Christoffel, und nach Eliminierung aus den beiden erhaltenen Gleichungen der Zwischenveränderlichen, erhält man unter Verwendung der Gleichung (2), die den Komplex Joukowsky's mit dem komplexen Potential

$$\Omega = w + iz$$

verbindet, die Lösung der Aufgabe in der Gestalt der Formel:

$$\operatorname{th}\left[\frac{\pi}{2T}\left(wi - z + \frac{s}{2}\right)\right] = k \operatorname{sn}\left[\left(-K + \frac{2K}{H} w\right), k\right] \quad (3)$$

worin $k = \operatorname{th}\frac{\pi s}{4T}$ — der Modul des elliptischen Integrals und K — der Wert des vollen Integrals erster Art für den Modul k ist *.

Die Formel für die Einheitsdurchflussmenge (wenn die Sickerungszahl k gleich 1 ist) hat die Form:

$$q = \frac{HK'}{2K} \quad (4)$$

wo K' den Wert des vollen elliptischen Integrals für den Ergänzungsmodul $k' = \sqrt{1-k^2}$ bezeichnet.

Die Gleichung der Sickerungskurve erhält man aus der allgemeinen Formel, wenn man $\varphi = y$, $\psi = 0$ und $w = y$ setzt.

$$\operatorname{th}\left[\frac{\pi}{2T}\left(\frac{s}{2} - x\right)\right] = k \operatorname{sn}\left[\left(-K + \frac{2K}{H} y\right); k\right]. \quad (5)$$

Die Gleichung für die Linie der wasserdichten Sohle erhält man bei $\psi = q$, $\varphi = y + T$

$$\operatorname{th}\left[\frac{\pi}{2T}\left(\frac{s}{2} - q - x\right)\right] = \operatorname{sn}\left[\left(-K + \frac{2Ky}{H} + \frac{2KT}{H}\right); k\right]. \quad (6)$$

Längs der Stromaustrittslinie im Unterwasser sind $\varphi = 0$, $y = 0$, und infolgedessen ist

$$\operatorname{th}\left[\frac{\pi}{2T}\left(\frac{s}{2} - \psi - x\right)\right] = \frac{-k}{\operatorname{dn}\left(\frac{2K\psi}{H}, k'\right)}. \quad (7)$$

Längs der Stromeintrittslinie ist

$$\operatorname{th}\left[\frac{\pi}{2T}\left(\frac{s}{2} - \psi - x\right)\right] = \frac{k}{\operatorname{dn}\left(\frac{2K\psi}{H}, k'\right)}. \quad (8)$$

Nach der Lösung der allgemeinen Formel bezüglich z erhält man durch Bildung der ersten Abteilung:

$$\frac{dz}{dw} = i - \frac{4KTk}{\pi H} \frac{\operatorname{cn}(\beta, k)}{\operatorname{dn}(\beta, k)} \quad (9)$$

wo $\beta = -K + \frac{2K}{H} w$ ist.

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* sn [(—), κ] ist die Bezeichnung für die erste elliptische Funktion Jakobi's.

Die allgemeine Formel für die Bestimmung der Geschwindigkeit hat die Form

$$\frac{dw}{dz} = v_x - iv_y = \frac{\mathrm{dn}(\beta, k)}{i\,\mathrm{dn}(\beta, k) - \frac{4KTk}{\pi H}\,\mathrm{cn}(\beta, k)} \tag{10}$$

insbesondere, längs der Ein- und Austrittslinien der Grundwasserströmung erhält man folgende Formeln:

$$v_y\,(\text{Eintritt}) = \frac{-\,\mathrm{cn}\left(\frac{2K\psi}{H},\ k'\right)}{\mathrm{cn}\left(\frac{2K\psi}{H},\ k'\right) + \frac{4KTk}{\pi H}\,\mathrm{sn}\left(\frac{2K\psi}{H},\ k'\right)} \tag{11}$$

und, beziehungsweise,

$$\frac{d\psi}{dx} = v_y\,(\text{Austritt}) = \frac{\mathrm{cn}\left(\frac{2K\psi}{H},\ k'\right)}{\frac{4KTk}{\pi H}\,\mathrm{sn}\left(\frac{2K\psi}{H},\ k'\right) - \mathrm{cn}\left(\frac{2K\psi}{H},\ k'\right)}. \tag{12}$$

Die nötige Tiefe der Einsenkung des Filters in den Bereich des Grundwasserstroms ergibt sich durch Bestimmung des Minimums für die Funktion $x = f(\psi)$ aus der Gleichung (12), d. h. unter der Bedingung, dass $\frac{dx}{d\psi} = 0$ ist.

$$\frac{4KTk}{\pi H}\,\mathrm{sn}\left(\frac{2K\psi}{H},\ k'\right) - \mathrm{cn}\left(\frac{2K\psi}{H},\ k'\right) = 0 \tag{13}$$

$$\psi_a = \int_0^{\sqrt{\frac{1}{1+b^2}}} \frac{d\omega}{\sqrt{(1+\omega^2)(1-k'^2\omega^2)}} \tag{13'}$$

wo $b = \frac{4KTk}{\pi H}$ ist.

Durch Einsetzen von ψ_a in die Formel (7) erhält man die Abszisse des linken Filterendes.

Zum Schluss mögen folgende Besonderheiten des vorliegenden Lösungsverfahren angemerkt werden:

1. Die Form der Oberfläche der undurchlässigen Sohle ergibt sich, wie auch die Sickerungskurve, infolge der Lösung. Dieser Umstand lässt die Aufgabe vom mathematischen Standpunkte aus viel einfacher erscheinen, als bei einer gegebenen horizontalen Schicht; zugleich stellt derselbe praktisch kein Mangel vor, sofern die erhaltene Sohlenlinie von der Horizontalen nicht mehr als um die Hälfte der Gefällshöhe abweicht.

2. Durch Erfahrung ist festgestellt worden, dass der Austrittspunkt der Sickerungskurve nicht immer mit der Unterwasserlinie am Dammkörper oder mit der Filteranfangsstelle, wenn der Filter keine Wasserschicht enthält, zusammenfällt. Infolgedessen können wir, streng genommen, die Lage dieses Punktes nicht ansetzen, bevor diese Erscheinung vollständig geklärt ist. In der obigen Aufgabe erhält man eigentlich den Austrittspunkt der Sickerungskurve bei der Lösung, und dadurch lässt sich eine der ungeprüften Annahmen vermeiden.

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PHYSICS

ON HYSTERESIS IN FERROMAGNETICS

By E. KONDORSKY

(Communicated by P. L. Kapitza, Member of the Academy, 9. XI. 1939)

The irreversible changes in magnetization responsible for hysteresis arise, according to modern views, from the existence of magnetic anisotropy which either is due to stresses or is a crystallographic one. They may be interpreted in terms of three factors:

1. After saturation has been reached and the field removed, there may be no magnetization embryos of the opposite sign and therefore no increase in the magnetization due to displacement of the boundaries between regions. The hysteresis loops which would be obtained, should the process of displacement of boundaries not take place, were examined in the works of Becker (1), Gans (2) and Akulov (3). Hysteresis of this kind should be attributed to an ideal ferromagnetic of infinite size and, possibly, also to a ferromagnetic with very fine and strongly deformed grains.

2. Delay may occur in the growth of the embryos of the new phase, owing to increase during this growth of the surface and surface energy of the boundary layer separating the embryo from the surrounding anti-saturated medium. This kind of hysteresis received its theoretical foundation in the work of Döring (4).

3. Displacement of the boundaries between different magnetic phases (regions) may be delayed, because in case of non-homogeneous material local variations are apt to occur either in the exchange integral (5) or in the magnitude of the internal stresses (6,7), on which the energy density of the phase and of the boundary layer depends.

Each of the three causes plays some part in the hysteresis of ferromagnetics. However, in a number of cases the hysteresis in the greater part of the volume of the ferromagnetic is due to only two or even one of these causes acting alone. For instance, if the coercive force of the specimen is, in all directions, of a smaller order of magnitude than the field intensity H_s, at which the magnetization I approaches saturation I_s, the hysteresis which takes place in the major part of the specimen's volume is certainly not due to the first cause. If, moreover, the difference between the maximum value of the magnetization (at the peak of the hysteresis loop) and the value of I_s is sufficiently great, a large part of the hysteresis losses in the major part of the volume is not due to the second cause either.

It has been shown by experiment that in the majority of cases the area of the hysteresis loop and the value of the coercive force do not depend on the shape of the specimen in polycrystalline ferromagnetics with uniform

magnetization. It will now be shown that in ferromagnetics with rectangular or almost rectangular loops this rule is, in general. not observed. Namely, if the hysteresis in ferromagnetics with rectangular loops is partly due to the second cause, or if it is due to the first cause (i. e. to the absence of embryos), the area of the hysteresis loop and the value of the coercive force depend on the shape of the specimen.

We shall demonstrate this by discussing separately the case when the hysteresis is due to the first cause, and the case when it is partly due to the second cause (being mainly due to the second and third).

a) The influence of the shape of the specimen on the coercive force in the absence of embryos of new phases with reverse magnetization (with hysteresis due to the first cause).

Let us suppose, for simplicity, that the direction of easy magnetization of an ellipsoidal ferromagnetic coincides with the principal axis of the ellipsoid and that the external field H_e is parallel to this axis. After saturation has been attained, changes in the longitudinal component of magnetization can occur in the case under consideration by no other means but through rotation of I_s. Let us consider two cases: 1) during the process of reverse magnetization the direction of I_s remains identical for all parts of the crystal, and 2) the direction of I_s may be different in different parts of the crystal, i. e. several magnetic phases may exist during the process of magnetization.

Case I. In this case the free energy density W_F is given by:

$$W_F = -H_e I_s \cos\vartheta + \frac{N_1}{2} I_s^2 \cos^2\vartheta + \frac{N_2}{2} I_s^2 \sin^2\vartheta + K \sin^2\vartheta \qquad (1)$$

where N_1 is the demagnetization factor of the ellipsoid in the direction of H_e, and N_2 — that in the direction of the longer axis perpendicular to H_e; ϑ is the angle between the directions of I_s and H_e. It is easily shown that W_F is a minimum when the longitudinal component of magnetization is:

$$I = -\frac{H_e}{\frac{2K}{I_s^2} - (N_1 - N_2)}. \qquad (2)$$

When $(N_1 - N_2) < \frac{2K}{I_s^2}$, hysteresis is present. In this case the coercive force $H_{c\,\mathrm{rot}} = \frac{2K}{I_s} - (N_1 - N_2) I_s$. As the length of the ellipsoid in the direction of H_e is reduced, the coercive force decreases. When $(N_1 - N_2) \gg \frac{2K}{I_s^2}$, the coercive force is zero and there is no hysteresis. The susceptibility of the body is then:

$$\chi_e = \frac{1}{N_1 - N_2 - \frac{2K}{I_s^2}}. \qquad (3)$$

Case 2. In this case ferromagnetic is split up into a number of phases during the process of reverse magnetization, so that the normal component of the magnetization becomes equal to zero. The free energy density W_F is then:

$$W_F = -H_e I_s \cos\vartheta + \frac{N_1}{2} I_s^2 \cos^2\vartheta + K \sin^2\vartheta. \qquad (4)$$

W_F is a minimum when:

$$I = -\frac{H_e}{\frac{2K}{I_s^2} - N_1}. \qquad (5)$$

The coercive force is equal to zero when $N_1 \gg \frac{2K}{I_s^2}$. However, hysteresis is not absent in this latter case, since when $H=0$ reversal of I_s takes place in only half the volume of the ferromagnetic. In the state when $I=0$, there exist two phases with spins in mutually opposed directions. As the field is increased, the increase in the magnetization takes place at the expense of the process of displacement. The initial susceptibility of the body is then equal to

$$\chi_{0e} = \frac{1}{N_1}. \tag{6}$$

b) The influence of the shape on the coercive force when the hysteresis is partly due to the second cause.

In this case reverse magnetization takes place as a result of an increase in the new phases at the expense of the old ones*. Döring (4) showed that ellipsoidal embryos of new phases can grow when the intensity of the effective field is

$$H \leqslant \left(H_0 + \frac{c_2\gamma}{4c_1 I_s d}\right) \tag{7}$$

where H_0 is the critical field, c_1 and c_2 are coefficients depending on the shape of the ellipsoidal embryo, d is its diameter and γ is the surface energy of the boundary layer. It follows from Döring's formula that if the demagnetization factor of the specimen N is other than zero, growth of the embryos is possible when the intensity of the external field is

$$H_e \leqslant -\left(H_0 + \frac{c_2\gamma}{4c_1 I_s d}\right) + NI. \tag{8}$$

The value of the magnetization I in the initial stages of embryo growth is related to the diameter of the embryos. If n is the embryo number per unit cross-section area of the ferromagnetic, then the magnetization

$$I = [1 - 2n\, c_1 d^2]\, I_s \tag{9}$$

whence

$$d = \sqrt{\frac{1 - \frac{I}{I_s}}{2nc_1}}. \tag{10}$$

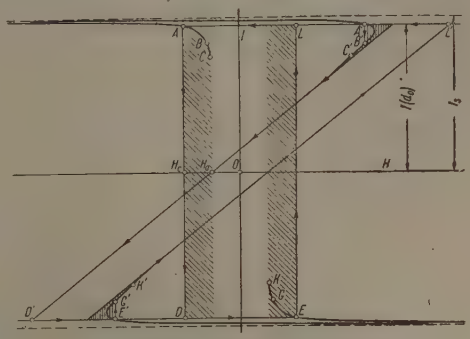
Substituting (10) into (8) and rejecting the inequality sign, we obtain an equation connecting H_e and I at the instant of the initial growth of the new phase:

$$H_e = -\left(H_0 + \frac{c_2}{4c_1}\sqrt{\frac{2nc_1}{1 - \frac{I}{I_s}}}\,\frac{\gamma}{I_s}\right) + NI. \tag{11}$$

* The presence of embryos of new phases is possibly due to the existence of spaces and flaws within the material. A space of ellipsoidal shape within a ferromagnetic magnetized to saturation forms a field equivalent to that of a dipole forming an obtuse angle with the direction of magnetization. If the principal axis of the ellipsoidal space coincides with the direction of magnetization, the field intensity H_i on the external surface of the space at points passed by its principal axis is $H_i = -I_s\, 4\pi\nu$, where ν is the demagnetization factor of an ellipsoid of the same shape as the space. After the external field is removed, if $|H_i| > \frac{2K}{I_s^2}$, there appears in the vicinity of the spaces (provided their transverse dimensions are considerably greater than the width of the boundary layer) a phase of reverse magnetization which may be an embryo capable of growth.

It is possible that the increase in the coercive force in very pure crystals of iron observed by Bozorth (8) and Williams (9) is a result of the reduction in the dimensions of the spaces included in the material. The circumstance that an increase in the coercive force was not observed in very pure permalloys is in favour of our supposition. Owing to the small values of the constants of magnetic anisotropy, in permalloy the value of the surface energy is considerably less than in iron. Therefore, the minimum value of the field, at which the growth of the largest embryos can still start, should also be less in permalloy.

Fig. 1 gives curves representing schematically the function $H(I)$ as found from formula (11). With an embryo of initial diameter d_0 and with $N=0$ only two stable states are possible, $I=\pm I(d_0)$. Transition from the one stable state to the other takes place at points A and D on the curve $H(I)$. The position of these points determines the width of the hysteresis loop. When $N>0$, points on the lines $B'C'$ and $G'K'$ are stable as are the points A' and D'. The part of the area of the hysteresis loop which depends on the second cause is hatched (Fig. 1). It will be obvious from an examination of Fig. 1 that the value of the coercive force depends on the initial diameter of the embryos and on the value of the demagnetization factor. When the initial diameter of the embryo d_0 is small in comparison with $\frac{1}{\sqrt{2n}}$ (i. e. $2nd_0^2 \ll 1$) and the demagnetization factor $N \ll \frac{c_2\gamma}{4c_1 I_s^2 d_0}$, we may consider to a first approximation that the magnetization $I(H_c)=I_s$. Then from (11) we obtain:



$N=0$ loop $ADEL$

$N > \frac{c_2\gamma}{4c_1 I_s^2 d_0}$ loop $A'B'C'D'E'G'K'L'$

$$|H_c| = H_0 + \frac{c_2\gamma}{4c_1 d_0 I_s} - NI_s. \qquad (12)$$

It is evident from (12) that H_c decreases as N is increased. When $N > N_{cr} \approx \frac{c_2\gamma}{4c_1 d_0 I_s^2}$ according to (12) $H_c \approx H_0$. It should, however, be noted that when $H_c = H_0$, the area of the hysteresis loop is greater than $4I_sH_0$ (Fig. 1).

If the difference $I_s - I_m$, where I_m is the maximum value of the magnetization in the internal loops, is sufficiently great, we have $H_c(I_m) \approx H_0(I_m)$. Therefore in those cases in which H_0 does not depend on the shape* of the specimen, the coercive forces $H_c(I_m)$ on the internal hysteresis loops should be independent of the shape of the specimen.

The influence of the shape on the coercive force in ferromagnetics with a rectangular hysteresis loop should be particularly strong, when the difference between the coercive force and the value of H_0 is of the same order as the value of H_0. Sixtus' ([10]) investigations showed that elastically extended permalloy is an example of this case; in this material the influence of the shape of the specimen on H_c is doubtless more accessible to investigation. If the assumption that in very pure closed crystals hysteresis is mainly due to the second cause is true, the value of the coercive forces on the internal hysteresis loops of these crystals should be very small compared with the value of the coercive force on the external loop. If we were to make a transverse section of closed crystals, the value of H_c on the external loops and on loops close to the exterior should be markedly reduced, whilst the value of the coercive forces on the inner loops should not change appreciably.

The following conclusions are to be drawn from the above statements: 1) if the area of the hysteresis loop and the coercive force of a ferromagnetic with an almost rectangular loop does not depend on the shape of the spe-

* See reference on page 742.

cimen, the hysteresis is not due to the second cause; neither is it due to the first cause; 2) the circumstance that the coercive force is independent of the shape for the majority of polycrystalline ferromagnetics should be explained either by the circumstance that hysteresis in these cases is not due to the first two causes*, or because the directions of easy magnetization do not coincide, and the values of γ are not the same in different parts of the specimen, so that the coercive force depends not on the shape of the ferromagnetic as a whole, but on the shape of sections of it, within which the values of γ and the directions of easy magnetization are unvarying. In both cases the unhomogeneousness of the material is responsible for the coercive force, being independent of the shape of the specimen.

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* It is easily verified that when hysteresis is due to the third cause, the reduction in the coercive force when N is increased can take place when the values of gradients $\frac{\partial \gamma}{\partial x}$ assume identical maxima close to the embryos and decrease as the distance from these latter increases. If the maxima of $\frac{\partial \gamma}{\partial x}$ are repeated a sufficiently large number of times in the spaces between the embryos, the reduction in the coercive force with increase in N either does not take place at all, or else is so small as to defy observation.

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CHIMIE

NOUVELLE MÉTHODE D'EXTRACTION ÉLECTROCHIMIQUE DE POLONIUM

Par D. M. ZIV

(Présenté par V. C. Chlopin, de l'Académie, le 3. XI. 1939)

Des méthodes connues actuellement d'extraction de polonium, l'une des plus usitées, pareillement à l'isolement de ce dernier par argent (1), est la méthode proposée en 1932 par O. Erbacher (2). Cette méthode consiste dans l'extraction de polonium par rotation d'une plaque de platine dans une solution de polonium pendant qu'un passage continu d'hydrogène a lieu. Dans ce cas la séparation s'effectue vite, donne un bon rendement, ne pose pas de limites à la densité superficielle du polonium extrait et représente une méthode commode pour la préparation suivante d'une solution pure de polonium. Néanmoins, tout en indiquant les avantages de la méthode, il convient de s'arrêter sur ses inconvénients.

1) Pour que l'extraction de polonium soit pratiquement totale, il faut l'effectuer dans un circuit rigoureusement fermé. La moindre trace d'air abaisse fortement l'effet. Ainsi, dans nos expériences d'extraction de polonium d'après cette méthode, le rendement ne dépasse pas 70—80%, si l'atmosphère d'hydrogène n'est pas isolée rigoureusement, tandis qu'avec l'emploi d'une fermeture hydraulique on obtient, dans des conditions identiques, un rendement de 95—96%, qui est en outre plus constant. Mais l'extraction de polonium devient un procédé plus difficile lorsqu'on emploie la fermeture.

2) L'hydrogène doit être suffisamment pur, car, dans le cas contraire, le rendement s'abaisse brusquement devenant encore très variable.

3) Le potentiel de l'électrode d'hydrogène, même dans un milieu fortement acide (+0.03 V pour 12% HCl), dépassant faiblement le potentiel Bi/Bi^{+++} à sa concentration ordinaire dans la source d'obtention de polonium—solution RaD, n'est pas en état d'empêcher le dépôt de RaE avec le polonium. Ce fait est fort fâcheux, de même qu'il n'est pas à désirer d'être obligé, dans ce cas, d'ajouter à la solution du bismuth.

En vue d'éviter les inconvénients mentionnés, tant au point de vue d'une certaine complication et finesse d'exécution, qu'au point de vue de la pureté du produit obtenu (absence de RaE), et d'autre part, en vue de conserver les avantages essentiels de la méthode citée, nous avons accompli un petit travail, ayant pour but de trouver une nouvelle méthode d'extraction de polonium, concordante à ces exigences.

Le potentiel critique d'extraction de polonium fut estimé en 1914 par Hevesy et Paneth (3) comme égal à + 0.63 V pour une concentration de $10^{-9}N$,

confirmé ensuite par plusieurs savants ([4–6]). Posant la valence du polonium dans la solution acide égale à +2, Hevesy et Paneth ont trouvé par extrapolation, suivant la loi de Nernst, que le potentiel normal Po/Po^{++} est égal à +0.85 V par rapport à l'électrode d'hydrogène.

Haissinsky ([7]), partant d'un potentiel d'extraction de polonium d'une solution de 10^{-9} *N* égal à $+0.64 \pm 0.02$ V et penchant à une valeur plus probable de valence dans solutions acides (+4), donne au potentiel normal Po/Po^{++++} une valeur égale à $+0.77 \pm 0.02$V suivant l'électrode d'hydrogène. Le potentiel normal Bi/Bi^{+++} est égal à +0.226 V ([8]).

Nous nous sommes servis des données indiquées en choisissant l'électrode la plus convenable pour l'extraction de polonium. A cet effet, c'est l'électrode quinone-hydroquinone qui convient, son potentiel dans la solution admise étant égal à +0.65 V. Les expériences que nous avons effectuées ont confirmé l'exactitude de notre choix et ont donné le moyen d'établir les conditions les plus avantageuses pour l'application de la méthode.

Presque dans toutes nos expériences, nous avons opéré de la manière suivante.

Légèrement traité par ébullition dans un mélange d'acide azotique concentré avec quelques gouttes d'eau oxygénée et ensuite dans l'acide chlorhydrique, le disque de platine de superficie de 4.5 cm^2 tournait à une vitesse de 160 t/min. dans une solution de volume de 3.5 cm^3. La solution était préparée, de préférence, par ébullition réitérée de tubes brisés d'émanation dans l'eau régale, évaporation à sec de l'extrait obtenu et dissolution du «résidu sec» dans HCl. On ajoutait à 3 cm^3 de solution de RaO, préparée de cette manière, 0.5 cm^3 de solution à 0.005 mol. de quinone-hydroquinone et d'acide chlorhydrique. A la fin de l'expérience on évaporait à sec la solution sur un verre de montre au bain-marie et on mesurait l'activité du «résidu sec» à l'aide d'un électroscope ordinaire α. On plaçait le disque sur un verre de montre identique et on mesurait l'activité de ses deux côtés. La quantité extraite de polonium était exprimée en pour-cent par rapport à la somme d'activités du disque et du résidu. Une division d'électroscope est égale à $1.13 \cdot 10^{-3}$ CGSE $= 0.93 \cdot 10^{-6}$ mCu Po $= 1 \cdot 10^{-12}$ mg-ions de polonium. La concentration moyenne de polonium dans la solution est $0.54 \cdot 10^{-10}$ mol/lit., lorsque l'activité moyenne est 186.4 div/min. Etant donné que le parcours des particules α du polonium est égal à 3.92 cm, et la couche résiduelle de quinone-hydroquinone, aprés évaporation, était égale à 0.12 mg/cm^2, la fraction des particules α absorbées par cette dernière était suffisamment petite (une pareille couche de quinone-hydroquinone équivaut approximativement à 0.09 cm d'air). En effet, la faute résultante de cette cause est petite en comparaison avec la faute d'expérience (tableau 1).

Tableau 1

Composition de la solution	Activité du «résidu sec»
Solution pure RaD	163.5 div/min.
Solution pure RaD + 3.3 mg quinone-hydroquinone . . .	169.6 » »
Solution pure RaD	11.1 » »
Solution pure RaD + 3.3 mg quinone-hydroquinone . . .	10.4 » »

On peut diviser les expériences exécutées en 3 séries: la première met en évidence l'intervalle minimum de temps nécessaire pour que la proportion de polonium, s'obtenant par extraction, devienne constante; la deuxième montre l'influence de la température; la troisiéme indique l'acidité la plus favorable de la solution. Les données principales de ces expériences sont incluses dans le tableau 2.

Il résulte du tableau donné:

1) La quantité déposée de polonium sur le disque de platine, en présence de quinone-hydroquinone, en partant d'une solution d'acidité de 0.1 *N* HCl

Tableau 2

No. par ordre	Série de l'expérience	Composition de la solution	Température	Durée de la rotation	Somme de l'activité du disque et du résidu, div/min.	Activité du résidu, div/min	Partie déposée de Po, %
1	1-ère	RaD; 0.1 *N* HCl; quin.-hydr.	Ambiante	30 m.	155.5	5.1	96.7
2	»	Dito	Ambiante	45 »	337.4	7.0	98.2
3	»	»	Ambiante	1 h.	428.0	12.1	97.2
4	»	Po; 0.1 *N* HCl; quin.-hydr.	Ambiante	1 »	183.8	4.0	97.8
5	»	RaD; 0.1 *N* HCl; quin.-hydr.	Ambiante	3 »	176.5	5.6	96.8
6	»	Dito	Ambiante	5 »	173.4	6.0	96.4
7	»	Po; 0.1 *N* HCl; quin.-hydr.	Ambiante	24 »	181,2	3.7	97.8
8	»	RaD; 0.1 *N* HCl; quin.-hydr.	Ambiante	3 »	11.3	2.0	82.0
9	»	Po; 0.1 *N* HCl; quin.-hydr.	Ambiante	2 »	55.3	9.6	82.7
10	2-me	RaD- 0.1 *N* HCl; quin.-hydr.	45°C	3 »	162.0	3.9	97.6
11	»	Dito	60°C	1 »	208.4	4.4	97.9
12	»	»	75°C	1 »	211.2	6.1	97.1
13	3-me	RaD; 1 *N* HCl; quin.-hydr.	Ambiante	1 »	183.5	50.3	72.7
14	»	RaD; 0.2 *N* HCl; quin.-hydr.	Ambiante	1 »	168.5	4.3	97.5
15	»	RaD; 0.02 *N* HCl; quin.-hydr.	Ambiante	30 m.	165.5	10.4	93.7
16	Ag-disque	RaD; 0.1 *N* HCl . . .	Ambiante	6 h. + * 24 h.	206.7	5.2	97.4
17	Ag-disque	Po; 0.1 *N* HCl	Ambiante	23 »	116.5	2.5	97.8

Erreur expérimentale = 0.5%.

et d'un volume de 3.5 cm³, est égale en moyenne à 97.3±0.5% et donne peu de variations hors de cette valeur. (En calculant la valeur moyenne du dépôt de polonium, on n'a pas tenu compte ni de l'expérience (8), l'erreur expérimentale relative étant assez grande, ni de l'expérience (9), où l'extraction s'effectuait en partant d'un volume de 16.5 cm³ et demandait apparemment beaucoup de temps.)

2) La température de la solution (dans nos limites) influe faiblement sur le résultat.

* Le disque, après une rotation pendant 6 heures, était immobilisé pendant 24 heures dans la solution.

3) L'acidité 0.1 *N*, admise dans les expériences, est la plus convenable.

4) Le temps suffisant pour atteindre un pour-cent constant de dépôt de polonium est égal à 30—45 min.

5) La quantité de polonium déposée sur platine, en présence de quinone-hydroquinone, équivaut à la quantité de polonium déposée sur l'argent dans les mêmes conditions, mais pour un temps beaucoup plus long.

L'absence dans le tableau 2 de colonnes concernant la conduite de RaE, lors du dépôt du polonium de RaD, s'explique par le fait que les concentrations faibles n'ont pas donné le moyen de déterminer avec une exactitude sufffisante l'activité de RaE sur le disque (activité RaE=2—2.5 div/min. avec une dispersion naturelle d'électroscope égale à 0.5 div/min.). Cependant, les expériences suivantes avec des solutions plus concentrées de RaD ont montré que le dépôt de polonium sur platine, en présence de quinone-hydroquinone, comme règle, n'est pas suivi de dépôt sensible de RaE, tandis que dans des conditions identiques on a un dépôt marqué de ce dernier sur l'argent (le potentiel Ag/Ag^+ dans nos conditions—0.1 *N* HCl—est sensiblement moindre que le potentiel d'électrode quinone-hydroquinone).

Comme résultat de notre travail, nous pouvons faire une série de remarques supplémentaires:

1) Le dépôt de Po sur Ag donne un rendement moindre et inconstant. Seule l'extraction réitérée nous conduit à des résultats identiques obtenus dans le cas du Pt par une seule rotation.

2) L'extraction du Po en présence de l'acide orthotellurique n'est pas suivie de dépôt de Te et donne un rendement de Po seulement un peu moindre que d'ordinaire (89.5%).

3) L'extraction de Po dans nos conditions a lieu jusqu'à l'établissement de l'équilibre, dépendant apparemment non pas de l'intersection des potentiels, mais seulement des causes cinétiques. Ainsi, pendant la rotation du disque de platine avec Po dans HCl pur, la quantité qui passe dans la solution est égale à la quantité extraite en pour-cent dans les expériences ordinaires.

4) La méthode que nous avons mise à point a l'inconvénient de porter une quantité de quinone-hydroquinone sous forme d'impureté dans la source d'obtention de Po. Cependant, en premier lieu, dans certains cas, par exemple en médicine, cela n'a pas d'importance, en second lieu, on peut diminuer sensiblement la quantité prise de quinone-hydroquinone, changeant peu de cette manière le potentiel de d'électrode, et en troisième lieu, on peut éliminer 99% de quinone-hydroquinone par simple évaporation avec HNO_3.

En conclusion, je tiens à remercier A. P. Ratner pour les indications précieuses qu'il a bien voulu me donner pendant l'exécution du travail exposé.

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GEOCHEMISTRY

ON THE NEUTRON RADIATION OF ROCKS

By N. M. LYATKOVSKAYA and G. V. GORSHKOV

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In the spring of 1938 Grammakov, Zhadin and the authors published a short article on the measurement of the intensity of neutron radiation of rocks with a silver counter (1). In the summer of 1938 neutron radiation of a radioactive ore was being measured by one of the recedent author in a factory (Gorshkov). The method of measurement and the apparatus were the same in both cases. In the process of these measurements a difference was discovered in the absorption of γ-rays by the iron and cadmium screens used which had been considered entirely equivalent.

In the winter of this year special experiments were carried out by Zhadin and one of us (Lyatkovskaya) as a partial check-up on the data published in 1938. These experiments consisted in the following: over a cast-iron screen including a paraffin screen and a silver counter there were placed successively at a distance of 60 cm from the center of the counter, an ampule of 0.3 mg Ra and one of 34 mCu Rn+Be, the latter being screened in addition by 10 cm of lead. The number of impulses in the silver counter was measured in both cases with the cadmium and iron screens of 1938 and with a newly prepared brass screen. A change of screens with the ampule Rn+Be was made every 20—30 min. The results of these measurements are set forth in Table 1.

Table 1

Effect with iron screen	Effect with cadmium screen	Effect with brass screen	Difference iron-cadmium	Difference brass-cadmium
Ampule with Ra				
$\frac{36373}{268} = 135.7 \pm 0.7$ impulses per minute	$\frac{39999}{309} = 129.4 \pm 0.7$	$\frac{37756}{296} = 127.6 \pm 0.7$	6.3 ± 1.4	-1.8 ± 1.3
Ampule with Rn + Be				
$\frac{31039}{128} = 250.3 \pm 1.4$	$\frac{24344}{180} = 135.2 \pm 0.8$	$\frac{34153}{150} = 231.2 \pm 1.2$	115.1 ± 2.2	$96.0 + 2$

It follows from the experiments that the effect of 6 impulses per minute which was obtained in 1938 with the ampule 0.3 mCu Rn+Be and was then attributed entirely to neutrons, is also obtainable with an ampule Ra of approximately the same strength. On the other hand, the data obtained with the ampule Rn+Be show that from 0.3 mCu Rn+Be we should have registered one extra «kick» per minute due to neutrons, even without taking into consideration the absorption and scattering of the neutrons by lead which was absent in the experiments of 1938.

From Table 1 we see, besides, that the difference in the effects for iron and brass increases approximately in proportion to the number of impulses in the counter, which undoubtedly points to the unequal absorption of γ-rays in iron and brass and in iron and cadmium, and not to the contamination of the iron screen by radioactive admixtures.

On the basis of these experiments we can conclude that the effect from radioactive ore measured in 1938 should be attributed at least in greater part to the unequal absorption of γ-rays in the iron and cadmium screens. The control experiment with and without cadmium carried out in 1938 is not decisive either, since in calculating the absorption, according to the King function, the difference in the effect between the number of impulses in the silver counter with and without a cadmium screen lies practically within the limits of experimental error.

A rough estimate which we omit for lack of space also shows that the effect of the neutron radiation of rocks should be tens of times less than that measured in 1938.

In connexion with this in the current year we carried out new experiments with an ionization chamber filled with BF_3 as an indicator of slow neutrons. The chamber was of steel with walls 3 mm thick. The diameter of the chamber was about 20 cm, the height about 28 cm. A Wulf bifilament electrometer was used.

Experiments on the measurements of the ionization current in a boron chamber filled successively with dry air free from emanation and BF_3 gas at atmospheric pressure and with various filtrations of γ-radiation showed, that the ionization in BF_3 by γ-rays of Ra is almost twice as high as in air, this ratio slightly increasing with an increase in the thickness of the filter. Further, the ionization from a preparation of Ra was measured in a Kolhörster counter and in a chamber filled with BF_3, in order to determine the comparative sensitiveness to γ-rays of Ra, since the Kolhörster counter was to serve as a control instrument in the measurement of neutron radiation in the atmosphere and in rocks. These measurements showed that the ionization by γ-rays in the boron chamber was 2.47 times higher than in the Kolhörster counter.

Finally, we made a quantitative comparison of the sensitiveness of the boron chamber and the silver counter to thermal neutrons. For this purpose an ampule Po+Be of approximate strength 0.3 mCu was placed in a paraffin box with walls 10 cm thick and an interior volume of $28 \times 28 \times 31$ cm^3 at a distance of 15.5 cm from the bottom and 17 cm from the middle of the box. The ionization chamber was placed in the middle of the box and closed on the top by a paraffin cover 10 cm thick; the ionization current was measured with borax and without. The ionization current with borax was equal to 43 ion pairs per cm^3 per sec.; without borax—106 ion pairs per cm^3 per sec. Thus, the neutrons caused an ionization current of 63 ion pairs per cm^3 per sec. With careful measurement we could have registered with the boron chamber a value one hundred times less. The silver counter under these conditions registered practically no effect at all. Upon substitution of the ampule

Po+Be by an ampule Rn+Be of the same strength the counter registered an effect of 11±2 «kicks» per minute.

Thus, the sensitiveness of the boron chamber made by us was considerably higher than that of the silver counter.

After these preliminary experiments we proceeded to the measurement of neutrons in the atmosphere. These experiments were carried out in a wooden house, built on weakly active soil. The ionization current was measured in boron chamber and in a Kolhörster counter with a borax screen and without one (in ion pairs per cm^3 per sec.). The results of these experiments are set forth in Table 2.

From this table it is clear that whereas in the Kolhörster counter within the limits of experimental error the ionization current with and without borax is the same, in the boron chamber the ionization current without borax is 1.8 ion pairs per cm^3 per sec. greater than with borax, which evidently should be attributed to the breaking up of borax nuclei by cosmic neutrons whose intensity is hereby determined as approximately one neutron per cm^2 per hour*. This effect in order of magnitude is in agreement with the data of Fünfer (2), Schopper (3) and Montgomery (4), but conflicts with the data of Halban (5) and others.

For an estimate of the intensity of neutron radiation of soil we used a rock in situ with an increased Ra content.

Table 2

Boron chamber		Kolhörster counter	
without borax	with borax	without borax	with borax
15.9±0.15	14.1±0.15	8.4±0.1	6.8±0.1

Table 3

Boron chamber		Kolhörster counter	
without borax	with borax	without borax	with borax
15.0±0.1 14.6	12.9±0.1 12.9	62±0.1	5.5±0.1

The first series of measurements of ionization currents in the boron chamber and in the Kolhörster chamber with and without borax was carried out in a clearing made in the rock in the form of a right prism. A rock of radioactivity approximately 50×10^{-12} Ra/g with a surface area of 100×150 cm^2 served as the lower base of the prism. Weakly active rock served as the three vertical sides of the prism with surface areas of 200×150 cm^2, $\frac{1}{2} \times 100 \times 200$ cm^2 and $\frac{1}{2} \times 100 \times 200$ cm^2. The instruments were placed successively on a cast-iron plate 12 cm thick with surface area 30×30 cm^2 in order to weaken the γ-radiation. The results obtained (in ion pairs per cm^3 per sec. minus the residual current and cosmic rays) are set forth in Table 3.

In the second column of the table are given the values for the boron chamber assuming that the absorption of γ-rays in borax is registered identically by the boron chamber and the Kolhörster counter.

From Table 3 it is evident that to the share of the cosmic neutrons+ the neutrons from the soil remain 0.4 ion pairs per cm^3 per sec., which contradicts the preceding measurements of cosmic neutrons in the wooden house. Therefore the boron chamber and the Kolhörster counter register the absorption of γ-rays in borax differently, which was to be expected a priori, since according to our preliminary measurements the boron chamber is more sensitive to hard γ-rays than the Kolhörster counter.

* The screening of the boron chamber by a paraffin sheet up to 10 cm thickness did not noticeably change the effect.

It is unfortunately impossible to account for this factor exactly, because of the complex composition of the radiation and our ignorance of the ionization functions of the apparatus. For this reason we carried out a second series of measurements in a similar clearing made in an active rock. In this case radiation entered the instruments not only from the bottom, but from the sides also. The activity of the rock was on the average also higher (about $100 \times^{-12}$ g Ra per g). On the other hand, the composition of the radiation remained approximately the same (broad beams of γ-rays, coming from various layers of the rock) so that the absorption law in borax should have remained approximately the same for both the boron chamber and the Kolhörster counter.

Table 4

Boron chamber		Kolhörster counter	
without borax	with borax	without borax	with borax
37.7±0.1 36.5	32.9±0.1 32.9	13.2±0.1	11.9±0.1

The data obtained (in ion pairs per cm^3 per sec. minus the cosmic rays and residual current) are given in Table 4.

In the second column, as in Table 3, are given the values for the boron chamber assuming that the absorption of γ-rays in borax is registered identically by both the boron chamber and the Kolhörster counter. Here to the part of the thermal neutrons already fall 1.2 ion pairs per cm^3 per sec., 0.8 ion pairs per cm^3 per sec. more than in the first case.

Thus, the intensity of neutron radiation of the soil which we studied * was evidently of the same order of magnitude as the intensity of cosmic neutrons at sea level. Therefore the intensity of neutron radiation of soil of normal activity (about 10^{-12} g Ra/g) should be tens of times less than the intensity of cosmic neutrons at sea level.

These measurements should be regarded simply as a check on the data of 1938. For a more accurate determination of the effect of the neutron radiation from rocks it is necessary to have a more sensitive apparatus.

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* It should be noted, that during the measurements there were frequent rains and the soil was damp.

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BIOCHEMISTRY

NUCLEIC ACIDS OF *ALLIUM CEPA* BULB

By A. N. BELOZERSKY

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It is mainly in seed sproots and reproductive cells that the nucleic acids of vegetable origin have been studied as yet. The results of the investigation have shown that almost in all cases two nucleic acids may be detected: yeast nucleic acid and thymonucleic acid. One of the first attempts at studying the nucleic acid in vegetative plant parts has revealed the developing buds of lime-tree to contain the thymonucleic acid, while the yeast nucleic acid was either absent there or found only in insignificant quantitites. Contrary to this, the cells of seed sproots contain rather large amounts of this latter type of nucleic acid, showing thus a different composition of their nucleic content compared to that in the lime buds. The high content of yeast nucleic acid in the cytoplasm of the cells of resting sprouts gives grounds for ascribing it a special rôle in the first phases of a plant's development from the seed. In the thesis by Ashkinazy, student of the Moscow University, it has been reported that 10-day old seedlings of soya still contain appreciable amounts of yeast nucleic acid. What is its further fate, should be cleared up by tracing its content at various phases of development. Another way to throw some light on the subject is to study the types of nucleic acids directly in vegetative parts of the plant.

In this connexion we arranged a test of cells of the bulb of *Allium cepa* for their content of nucleic acids.

Since the nuclei of cells of *Allium cepa* had been reported to stain according to Feulgen [2], we made an attempt at obtaining thymonucleic acid. The method of Neumann was employed for the purpose. Its advantage was in that boiling with strong alkali should decompose yeast nucleic acid which might be present in the onion cells. 3.5 g of substance containing 2.5 % phosphorus and 3.8% nitrogen was obtained. The biuret reaction of this product was negative, and a considerable amount of carbohydrate admixtures was found in it. Thymin was isolated and identified on its hydrolysis. This fact left no doubt as to the presence of thymonucleic acid in the product. Its amount might be estimated as 25—30%, but it could not be freed from the carbohydrate admixtures.

The question as to the presence of yeast nucleic acid in cells of *Allium cepa* was attacked by hydrolyzing the whole onion, whereby the products of hydrolysis were investigated. 1500 g of onion bulbs were taken for this test. Previous to hydrolysis the onion was fixed in Koch's apparatus, shred and set free from soluble carbohydrates and salts by way of dialysis. The

hydrolysis was run with 25% H_2SO_4 during 24 hours. From the hydrolysate were isolated and identified adenine (melting point of picrate—279° C), cytosine (after decomposition of picrate, Weeler-Johnson's reaction) and thymin (about 20 mg, N=22.15%). We failed to isolate guanin, although the reaction for this substance of the respective fraction was positive. This is probably because guanin, as shown by numerous observations, is readily broken down when impure preparations are hydrolyzed. Neither in the crystalline precipitates, nor in the mother liquor did we obtain any traces of uracil, this characteristic product of yeast nucleic acid breakdown.

It is therefore only thymonucleic acid that has been detected in the cells of bulbs of *Allium cepa* by way of hydrolysis. The fact that no products characterizing the yeast nucleic acid was found in the hydrolysate might be taken to mean that this acid is either entirely absent from the *A. cepa* cells, or available in insufficient quantity to be detected by the usual indirect methods of hydrolysis. Further proofs, however, were needed to put this beyond doubt. For it has been established through special experiments, that sodium salt of yeast nucleic acid is perfectly dialyzable, and our *A. cepa* being dialyzed before hydrolysis, there is a strong possibility of the yeast nucleic acid being lost in that way, the more so if it was not held in the cells by protein substances.

The conclusion to be drawn from the tabulated data on the estimation of nitrogen forms in the hydrolysate is, that the main groups fail to predominate in the bulk of protein substances available in *A. cepa* cells, and the latter with regard to their composition approach to the ordinary vegetable proteins.

Nitrogen Forms in Hydrolysate

Forms of N	N in g	N in g
Total N	2.625	100
N of ammoniac	0.124	4.72
N of bases	0.602	22.93
N of monoaminoacids	1.380	52.56

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MINERALOGY

LOVOZERITE—A NEW MINERAL

By V. I. GERASSIMOVSKY

(Communicated by A. E. Fersman, Member of the Academy, 5. XI. 1939)

This mineral was found by the author in the summer of 1935 in the Lovozero alkaline massif located in the central part of the Kola peninsula. In the course of examination it was found to be unidentifiable with any mineral already known. Therefore, it has been named lovozerite, after its finding place.

Physical Properties. Lovozerite in crystals has not been encountered. It has been observed in grains up to 1—2 cm in size. The mineral is black, frequently dark-brown owing to an abundance of small inclusions of other minerals. Streak brown. Lustre resinous. Opaque. Cleavage not observed macroscopically. Fracture from uneven to conchoidal. Hardness about 5. Specific gravity as determined by means of a pycnometer 2.384.

Under a blowpipe easily melted into an opaque ball of white colour with a pink tinge. With borax in an oxidizing and reducing flame yields a bead of light-violet colour which is preserved after cooling. A phosphate bead in an oxidizing and reducing flame is greenish-yellow when hot, and almost colourless when cold.

Optical Properties. Optically lovozerite is uniaxial, negative. Refractive indices (as determined by immersion) $Nm=1.561$ and $Np=1.549$. Cleavage imperfect, rarely observed, corresponding to Nm. Twins, not infrequently polysynthetic, are often observed in lovozerite. In twins Np of one individual with Np of the other forms an angle of 73—74°. The pole of the twinning suture with Np of both individuals shows one and the same angle of 36—37°. Colour in thin-section light-pink with a slight yellowish tinge. Pleochroism poorly expressed. Parallel to Np a slight deepening of the pink colour is observed. The colour of the mineral along cracks (frequently observed in lovozerite) and at the grain borders is usually light-pink with a brownish tinge, brown or rusty-brown. In the latter case the mineral in thin-section is frequently opaque. At the grain borders polyzonality is occasionally observed, manifested in that the outer border of the grains is usually brown and frequently isotropic. In one of the lovozerite grains in a section $\perp Np$ zonality was observed along the hexagonal prism. Lovozerite usually contains abundant inclusions of other minerals, chiefly nepheline, K-feldspar, aegirite, etc.

The above characteristics of physical and optical properties refers to lovozerite observed in large grains. Besides, lovozerite is encountered in small grains, rarely exceeding 0.03 to 0.3 mm in size. Small lovozerite grains are transparent, unbroken, show light-pink colouring without brown tinges,

and contain no inclusions of other minerals. Twins are poorly expressed, showing mostly either wavy extinction, or else consisting in their turn of still smaller individuals which are not simultaneously extinguished when examined in thin-sections.

X-Ray Analysis. In order to determine the syngony of lovozerite the latter was subjected to X-ray examination which proved unsuccessful. The photographs were made by the Debye method (Fe-rays) three times, and only once when the exposure was about seven hours the film showed a few very weak lines resembling Debye lines.

Chemical Properties. Lovozerite is insoluble in acids—HCl, HNO_3 and H_2SO_4. Its chemical analysis has been made by T. A. Burova (deposit in the upper part of the Muruai river). The results are given in the table.

Chemical Analyses of Lovozerite, Eudialyte, Mesodialyte and Elpidite

Oxides	Lovozerite		Eudialyte % *	Mesodialyte % *	Elpidite % *
	%	equiv. quant.			
SiO_2	52.12	0.868	49.16	51.23	59.44
TiO_2	1.02	0.013	0.72	0.56	traces
ZrO_2	16.54	0.134	14.17	13.98	20.48
TR_2O_3	0.56	0.002	2.29	1.68	—
Al_2O_3	0.40	0.004	—	—	—
Fe_2O_3	0.72	0.005	0.78	0.67	—
FeO	—	—	3.57	3.39	0.14
MnO	3.46	0.049	1.64	2.50	—
MgO	0.76	0.019	—	2.27	—
CaO	3.34	0.059	7.01	9.62	0.17
SrO	0.06	0.001	1.42	—	—
Na_2O	3.74	0.060	15.82	11.16	10.41
K_2O	1.90	0.020	0.43	0.69	—
$H_2O^{+110°}$	8.62	0.479	1.26	2.88	5.72
$H^2O^{-110°}$	6.41	0.356	—		3.89
Cl	none	—	2.19	1.26	0.15
	99.65	—	100.46	99.98	100.53
$-O=Cl_2$	—	—	0.49	0.27	—
Total	99.65	—	99.97	99.61	100.53

According to the data of the chemical analysis, the empiric formula of lovozerite is as follows: $(H,Na,K)_2O.(Ca,Mn,Mg)O.(Zr,Ti)_2O.6SiO_2.3H_2O$.

Lovozerite is to be referred to the group of Zr silicates. Its chemical composition points to a closest relation to eudialyte (and its varieties, mesodialyte and elpidite). Their analyses are given for comparison in the table. Spectroscopic examination carried out by S. A. Borovick (Institute of Geological Sciences of the Academy of Sciences of the USSR) showed it to contain Be and Hf (weak lines), beside the elements found by chemical analysis. Qualitative X-ray chemical analysis of lovozerite made by I. B. Borovsky (Institute of Geological Sciences of the Academy of Sciences of the USSR) showed that apart from the elements detected by chemical analyses it contains Hf~0.1% and U~0.2%.

* Analyses taken from the book «Minerals of the Chibina and Lovozero Tundras», 1937. Published by the Academy of Sciences of the USSR, p. 298 and 304.

Occurrence and Paragenesis. Lovozerite in the Lovozero alkaline massif is one of the rock-forming or secondary minerals of certain varieties of porphyry-like luyavrite. The latter are most widely spread in the northern and western parts of the massif. The fine-grained groundmass of porphyry-like luyavrites containing lovozerite consists of microcline, nepheline and aegirite. The following secondary minerals have been observed: eudialyte, lovozerite, lamprophyllite, sodalite, albite, zeolite and occasionally neptunite. Of these eudialyte and lovozerite are frequently found in considerable quantities. The zeolites and probably all the sodalite are secondary minerals after nepheline, while albite is secondary after microcline. Phenocrysts in porphyry-like luyavrites containing lovozerite consist of: lovozerite, murmanite, lamprophyllite, alkaline amphibole and nepheline. Of these lovozerite and murmanite are usually quantitatively prevalent. The content of lovozerite in porphyry-like luyavrites sometimes amounts to 10—20%.

Lovozerite is of various genesis. It may probably be of primary and secondary origin. When observed in large grains, lovozerite is to be regarded in all probability as a primary mineral crystallized together with murmanite (hydrous niobo-titano-silicate) towards the end of the formation of porphyry-like luyavrite, at the expense of residual magma considerably enriched with water and probably relatively impoverished in alkalies. The latter were largely spent for formation of nepheline, microcline and aegirite which crystallized (possibly except aegirite) previous to lovozerite. The primary formation of lovozerite encountered in large grains is evidenced by the following data: 1) the presence in lovozerite of a large number of inclusions of other minerals (microcline, nepheline, sometimes lamprophyllite, aegirite, etc.) which are usually absent in eudialyte; 2) the outlines of separate lovozerite grains are usually uneven and sometimes sinuous, while those of large eudialyte grains substituted with lovozerite are even, and 3) under the microscope large lovozerite grains become extinguished simultaneously or in accordance with the twinning observed in it.

Lovozerite observed in small grains is a secondary mineral—after eudialyte. This is shown by the following:

1. In thin-sections there may be frequently observed substitution of lovozerite for eudialyte.

2. The shape and size of small lovozerite grains and of the associated non-substituted eudialyte are similar.

3. Small lovozerite grains not infrequently consist of minute individual aggregates which are extinguished not at the same time.

4. Eudialyte contains no inclusions of other minerals, and the lovozerite formed after it must not contain such. The absence of inclusions of other minerals in small lovozerite grains (these inclusions always being found in large lovozerite grains) is also an indirect evidence of the secondary formation of lovozerite (in small grains).

Aside from this it should be noted that the postmagmatic processes in porphyry-like luyavrites are manifested alongside of the substitution of lovozerite for eudialyte also in the albitization of microcline, in the formation of sodalite and zeolites after nepheline.

Lovozerite differs from eudialyte in that it contains less alkalies and more water. On account of this and of the fact that secondary processes took part in the formation of porphyry-like luyavrites, the conclusion suggests itself that in the course of crystallization of primary lovozerite the conditions for the previously crystallized eudialyte were unfavourable, and the latter was substituted with lovozerite. Thus, along with crystallization of primary lovozerite, the formation of secondary lovozerite (after eudialyte) took

place simultaneously with and possibly also subsequent to this process.

The discovery in the Lovozero alkaline massif of a new mineral—lovozerite (hydrous zirconium silicate) is a fact of great interest. So far it has been known that in the rocks composing this massif along with anhydrous titanium silicates (ramsayite, loparite, lamprophyllite) a hydrous titanium silicate—murmanite—was encountered which has been formed later. It was natural to suppose that an analogous phenomenon would be observed among the zirconium silicates, too. This assumption was justified by the finding of lovozerite—a hydrous Zr silicate—along with an anhydrous Zr silicate—eudialyte. According to the time of formation, lovozerite is a later mineral than eudialyte and is encountered together with murmanite.

The discovery of secondary lovozerite still more emphasizes the important part played by postmagmatic processes in the formation of the Lovozero alkaline massif, which have been manifested in the substitution of sodalite and zeolites for nepheline, of ussingite and zeolites for sodalite, of astrophyllite for aenigmatite, of albite and ussingite for microcline. It is to be noted, moreover, that the part played by water was considerable.

Taking into account the enormous reserve of eudyalite, economic utilization of lovozerite can scarcely be discussed until due research and prospecting work is carried out.

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STRATIGRAPHY

ON THE STRATIGRAPHIC VALUE OF THE UPPER CARBONIFEROUS AND LOWER PERMIAN BRACHIOPODS

By D. L. STEPANOV

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In spite of the fact that brachiopods represent one of the most frequently encountered groups of fossils in Upper Carboniferous and Lower Permian deposits, their stratigraphic significance has not yet been sufficiently elucidated.

While it is long since the stratigraphic value of this group had been firmly established both for the Devonian and the Lower Carboniferous and was lately estimated for the Middle Carboniferous, too, the question as to the significance of brachiopods for the stratigraphy of the Upper Carboniferous and the Permian remains to a considerable extent open to discussion.

On the one hand, we know authoritative advocates of the stratigraphical value of Upper Paleozoic brachiopods. To these, as it is known, belongs F. N. Chernyshev. A. Grabau, Ch. Schuchert and H. Frebold are making wide use of brachiopods to correlate sections of the Upper Paleozoic.

On the other hand, in this country one seems to be somewhat disappointed in the Upper Paleozoic brachiopods as indicators of age. It is commonly assumed that brachiopod fauna suffered no marked changes during the Upper Carboniferous and the Lower Permian, and that separate complexes of this fauna show rather facial than age differences.

We believe these doubts concerning the stratigraphic value of brachiopods to be largely due to the confusion which prevailed in the stratigraphy of the Upper Paleozoic until recently and begins to dissipate now.

The author was engaged during several years in a study of problems of stratigraphy on the brachiopod fauna of the Upper Paleozoic. From an analysis of the literary data available and a treatment of his own rather abundant material, he arrived at a conclusion that some complexes of brachiopods from the Upper Carboniferous and Lower Permian deposits of the southern part of the USSR can be tentatively assigned to characterize definite stratigraphic horizon.

G z h e l C o m p l e x. A complex of brachiopod fauna designated by the author as the Gzhel complex characterizes the lower part of the Upper Carboniferous—Lower Permian beds, overlying directly the Middle Carboniferous and corresponding approximately to the *Triticites* zone.

The following three chief elements may be clearly distinguished in the Gzhel complex. The first—a group of forms, native only to the given complex and not passing to higher horizons. To these are referred first of all number of species of the subgenus *Choristites*, represented by coarsely costate forms,

known in literature under the common denomination of «the Samara forms». Of the *Productidae* the following are particularly characteristic: *Productus gratiodentalis* Grabau, *Pr. subpunctatus* Nikit., *Pr. pseudoarticnsis* Stuck. with their varieties. To these are usually added representatives of the subgenus *Dictyoclostus*, of which most characteristical are the forms belonging to the group *Pr. gruenewaldti*, but distinguished from the typical representatives of the Krotov species by small dimensions and certain peculiarities of sculpture. Similar forms have been described under different names from different places, but all of them represent, evidently, only vicar species or varieties grouping themselves about *Pr. bathycolpos* Schellw.

The second element of the Gzhel complex is an extensive group of forms which appear there for the first time, passing also to higher horizons. Among the most common forms of this group let us quote *Orthotichia morganiana* Derby, *Chonetes uraticus* Moell.; *Pr. pustulatus* Keys.; *Pr. stertitamakensis* Step., *Spirifer poststriatus* Nikit., *Sp. rectangulus* Kut., representatives of the genera *Camarophoria* and *Ilustedia*, etc. It is in the Gzhel complex also that appear the first *Spiriferella**, though represented by forms differing from the group *Sp. saranae* Vern. characterizing only higher horizons. These are minute forms with simple costae. Among these is numbered *Sp. gjeliensis* sp. nov. (=*Sp. saranae* Nikitin non Verneuil).

It is of interest to note the similarity of evolution of *Spiriferella* in the Upper Paleozoic of North America. In Pensylvania this genus is represented by *Sp. texana* Meek**—a form closely approaching to our Upper Carboniferous *Spiriferella* and not encountered beyond the formation Volfkamp. Large *Spiriferellae* of the group *Sp. saranae* (*Sp. sulcifer* Schum.) appear only in Permian deposits, being especially characteristic of the formation Vord.

The last element present in the Gzhel complex is a number of forms of a Middle Carboniferous habit—*Enteletes* ex gr. *tanarkii* Fisch., *Chonetes carboniferus* Keys., *Choristites* ex gr. *mosquensis* Fisch. and some others.

The Gzhel complex characterizes the *Omphalotrochus* and *Fusulinella* horizons of the Moscow basin, a horizon with *Spirifer jugulensis* of Samarskaya Luka, the Panikian series of the region along the lower part of the Volga river, the Isaevo, Avilovo and Araukarita series of the Donetz basin. At the Urals the Gzhel complex characterizes the Zilim beds separated by G. I. Teodorovich (²) in Mountainous Bashkiria. Elements of the Gzhel complex are present in the Auernig beds of the Carnic Alps, in the Upper Carboniferous of Ferghana and in the Taiyuen series of North China.

Uralian Complex corresponds to the zones *Pseudoschwagerina*, *Schwagerina moelleri* Schellw. and *Schw. anderssoni* Schellw. This complex is distinguished by its abundance and variety, especially in reef facies.

Only the most characteristic forms are listed here: *Productus genuinus* Kut., *Pr. kutorgai* Tschern., *Pr. transversalis* Tschern., *Pr. pseudomedusa* Tschern., *Pr. jakowlewi* Tschern., *Pr. simensis* Tschern., *Pr. tuberculatus* Moell., *Pr. tuberculatiformis* Frcks., *Pr. neoplicatilis* nom. nov. (=*Pr. plicatiliformis* Frks. non Demanet), *P. juresanensis* Tschern., *Spirifer tyra* Kut., *Sp. kotzapfeli* Tschern., some species *Pseudomartinia* and *Martiniopsis*, «*Rhynchonella*» *granulum* Eichw., *Septacamera krotovi* Tschern., *S. hofmanni* Krot., a number of species *Wellerella*, *Pugnax*, etc. In the Uralian complex evidently first appear the genera *Cryptacanthia*, *Keyserlingina*, *Aubosteges*.

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* We think that the Lower Carboniferous forms of the type *Sp. plena* Hall. referred to *Spiriferella* are so different from the genotype, that their belonging to this genus is rather doubtful.

** To judge by the description and drawings by Dunbar and Condra (¹), the latter undoubtedly belongs to *Spiriferella*.

Most probably, later on it will be possible to subdivide the Uralian complex into two subcomplexes. The lower one corresponding to the *Pseudoschwagerina* zone will be characterized by the above list, while in the upper subcomplex to this list will be added a number of forms, passing to higher horizons: *Productus uralensis* Lich. (=*Pr. uralicus* Tschern. 1902), *Pr. porrectus* Kut., *Pr. irginae* Stuck., *Pr. gruenewaldti* Krot., *Spirifer nikitini* Tschern., etc.

The Uralian complex is widely spread on the western slope of the Urals. It is represented in the limestones of the Kozhim-Terovei river, in the *Pseudoschwagerina* limestones of the region between the Kama and Pechora rivers, in the lower horizons of the calcareous beds of the Ufa plateau, in the Sterlitamak monad rocks. A similar brachiopod complex characterizes the Maping series of South China.

Artinskian Complex nearly corresponds to the zone *Parafusulina lutugini* Schellw. It differs from the preceding one, first of all, by the disappearance of a number of Uralian forms, owing to which the specific composition of the Artinskian complex, even in the reef facies, is impoverished compared to that of the Uralian complex. At the same time it is here that first appear a number of very characteristic forms absent in lower horizons. To these, first of all, should be referred the group *Spiriferella saranae* Vern.

Owing to their cosmopolitan distribution and frequent occurrence, the representatives of this group occupy the first place among the leading forms of the Lower Permian. By its stratigraphic value this group may be compared with the *Pseudoschwagerina*.

Of the *Productidae* the following forms are worth mentioning as being characteristic of the Artinskian complex: *Productus stuckenbergi* Krot., *Pr. pseudoaculeatus* Krot., *Pr. orientalis* Frcks., *Pr. ufensis* Frcks., *Pr. artiensis* Tschern., *Pr. mammatiformis* Frcks., *Pr. sylveanus* Stuck., and a number of *Chonetes* (*Chonetina sinuata* Krot., *Chonetes solidus* Krot., etc.).

Aside from these some species of *Productidae* are characteristic appearing already in the Uralian complex, but flourishing only in the Artinskian where they are encountered in great abundance. These are: *Productus gruenewaldti* Krot., *Pr. uralensis* Lich., *Pr. aagardi* Toula, *Pr. irginae* Stuck.

This complex characterizes the terrigenous «Upper Artinskian» deposits of the whole western slope of the Urals. In calcareous facies it is present in the upper horizons of the Ufa plateau (Irgin, Sargin and Krasnoufimsk), in the limestones of Boyets at the Kolva river (North Urals), in the *Cladochonus* limestones of Bashkiria.

Kungurian Complex is the less clearly outlined, which depends on the vagueness of the very idea of the Kungurian stage as a stratigraphical unit, as well as on the facial peculiarity of the deposits usually referred to the Kungurian (sediments of a basin with unusual salinity). With the latter circumstance is connected to a considerable extent also the relict character of the Kungurian brachiopod fauna, among which prevail certain forms of the Artinskian complex: *Spiriferella* ex gr. *saranae* Vern., *Productus irginae* Stuck., *Pr. septentrionalis* Tschern. var. *disjuncta* Step. A peculiar feature of the Kungurian complex is the first appearance of a zechstein element represented by *Productus cancrini* Vern., *Pr. pseudotimanicus* Ger., *Attyris royssiana* Keys., *Spirifer* aff. *rugulatus* Kut. If the author is right in admitting the Kungurian age of the *Spirifera* and *Productae* beds in Spitzbergen and in the upper part of the sub-coal-bearing beds of the Polar Urals, *Productus borealis* Haught. and *Pr. arcticus* Whitf. are also to be numbered among the characteristic forms of the Kungurian complex.

Thus, at present it is possible to separate four or five brachiopod complexes characterizing definite horizons of the Upper Carboniferous and the Lower Permian of the European part of the USSR. Throughout the same section of the stratigraphic column *Ammoneae*, according to Ruzhenzev [3], and bryozoans, according to Nikiforova [4], give only three complexes each. Besides, in contradistinction to brachiopods, neither of these groups shows universal distribution, being connected with definite facies. *Fusulinides* can be subjected to very detailed subdivision in separate regions (up to 12 zones within C_3 and P_1). Part of these zones is, however, only of local importance, and not more than four or five zones are to be regarded as universal and having analogues also in brachiopod complexes. It should therefore be stated that in a number of cases the degree of accuracy of subdivision of the Upper Paleozoic attained with the aid of brachiopods is not inferior to the stratigraphy based on such well-tried groups as *Fusulinides* and *Ammoneae*.

If, moreover, it be considered that the definition of the guide forms of brachiopods is possible under field conditions, too, it becomes evident that the given group may be of serious importance as a stratigraphic correlative and from this standpoint it deserves more attention than it was given heretofore.

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STRATIGRAPHY

ON THE FRESH-WATER HORIZON BETWEEN THE PONTIAN AND THE MAEOTIC DEPOSITS OF THE SOUTH OF THE UKRAINIAN SSR

By I. J. YATZKO

(Communicated by A. A. Borissiak, Member of the Academy, 3. XI. 1939)

In 1897 Sintzov (9) described peculiar *Unionidae* from different places of the present Odessa region. The age of the beds, containing the *Unionidae* fauna described by Sintzov, is referred by the author to Maeotic and partly Upper Sarmatian. Later on, an analogous fauna was encountered by other investigators, the area of distribution of the fauna spreading with new findings. The stratigraphic position of the findings was determined by Sintzov as the Maeotic—Upper Sarmatian.

By 1939 we have the following data on the specific composition and distribution of the *Unionidae* fauna in the southern parts of the Ukrainian SSR (table).

In 1936 was published a paper by Osaulenko (4). The author dwells at great length upon the mode of occurrence of the *Unionidae* fauna and places the latter in the lower part of the Maeotic and the upper part of the Upper Sarmatian. The author reports new localities of occurrence of the fauna discussed.

The facts that were at our disposal conflicting with the inferences of Osaulenko, have induced us to undertake a number of excursions for collecting *Unionidae* fauna and obtaining additional data concerning its mode of occurrence. As a result, a large paleontological material has been collected which we have treated in detail.

The results of the treatment permit of the following conclusions to be suggested:

1. The spread of deposits of the *Unionidae* fauna within the Ukrainian SSR is confined by the following points: the northernmost and at the same time easternmost point—the village of Bereznegovatoye (the Visun river, right tributary of the Inguletz river); the most western point—the village of Grebenniki, and the southernmost one—the town of Odessa.

2. The deposits containing the *Unionidae* fauna are represented mainly by fine-grained clayey sands, sometimes clays. These deposits are always cross-bedded, rarely contain intercalations of larger components.

3. The occurrence of the *Unionidae* fauna is characterized by conditions somewhat different from those described in literature. The study of exposures in the village of N. Bogdanovka where, according to Osaulenko (5), the *Unionidae* fauna is overlain by limestone with *Dosinia maeotica* and *Cerithium*, justifies the introduction of greater precision into the data communicat-

Name of species	Locality of occurrence	Reference	Age of fauna according to the author's data
Unio flabelatus Goldf.	Ananiev, Gandrebura, Lipetzkoye, Syrovo	6	Maeotic
	Kordon, Belyary, Shirokoye	2	»
	N. Elizavetovka, Grebenniki	1	Maeotic and Upper Sarmatian
	Odessa (environs)	8	Maeotic
	Grebenniki	3	»
Unio novorossicum Sinz.	N. Bogdanovka	9	»
	Broshevanovka, Prishib	4	»
	Ananiev, Syrovo	6	»
Unio radiato-dentatus Sinz.	N. Bogdanovka, Kualnitz liman	9	»
	Bereznegovatoye	4	»
	Tzebrikovo	1	»
	Pshenyakovo	2	»
Unio partschi Pen.	Broshevanovka, Neusatz	9	»
	Varvarovka	7	»
	Prishib	4	»
	Ananiev, Gandrebura, Syrovo	6	»
	Pshenyakovo, Belyary	2	»
	N. Elizavetovka, Pivovarovo	1	Maeotic, Upper Sarmatian
	Grebenniki	3	Maeotic
Unio sub-hörnesi Sinz.	Neusatz, Varvarovka, Broshevanovka, Ananiev	9	»
	Pshenyakovo, Belyary, Kordon	2	»
	N. Elizavetovka, Pivovarovo, Grebenniki, Tzebrikovo	1	»
	Grebenniki	3	»
Unio subrecurvus Teiss.	Grebenniki	3	»
Unio atavus Partsch.	Varvarovka, Neusatz, Belyary,		
Unio neumayeri Pen.	N. Bogdanovka	7	»
Vivipara novorossica Sinz.	Metleshevka, Broshevanovka	9	»
	Prishib, Snegirevka	4	Maeotic, Upper Sarmatian
	Sirotskoye, Pivovarovo, Tzebrikovo	1	Maeotic
	Dalnik, Stepovka	10	»
Vivipara barbori Sinz.	Prishib	4	Maeotic, Upper Sarmatian
Vivipara bifarcinata	Prishib	10	Maeotic

ed by Osaulenko. It is found that the Maeotic limestone lies over beds and among beds with *Unionidae* fauna not as a continuous horizon, but as blocks of various sizes, bearing traces of being water-rounded, i. e. blocks not in situ. The examination of many exposures in nature and their study according to literary sources shows the absence of the *Unionidae* beds being overlain by paleontologically proved Maeotic rocks. The geological section near Odessa shows that the *Unionidae* beds are overlain by Pontian deposits and rest on clays with *Dosinia maeotica* and *Cerithium disjunctoides* Sinz. The exposures in the villages of Prishib (4), Varvarovka and others (10) show that the *Unionidae* fauna occurs also on Upper Sarmatian limestones, being accompanied with poorly water-worn limestone pebbles.

Thus, it may be considered as established that the deposits containing the fauna of *Unionidae* lie between Pontian deposits and Maeotic deposits with *Dosinia maeotica* and *Cerith. disjunctoides* Sinz. and rest either upon the latter, or upon eroded Upper Sarmatian rocks.

4. The determination of the specific composition of the *Unionidae* fauna according to our collections, as well as on the basis of the collections of

Sintzov, Alexeev, Krokos and Gaponov, which were at our disposal, permits of the following statements to be made:

a) In the collections studied by us, typical *Unio flabelatus* Goldf. known from literature are absent. The presence of this species in the lists of the preceding investigators is accounted for by inaccurate determination, which by the way had been suspected as early as by Laskarev (3). As a matter of fact, one has to do here with *Unio wetzleri* Hörn.

b) Smooth species of *Unio*, named by Sintzov *Unio novorossicus* and *U. sub-hörnesi*, do not differ at all from the species described by Peneke (13) under the name of *Unio neumayeri*, i. e. the names given by Sintzov should be considered nothing but synonyms of those given by Peneke.

c) Along with shells of *Unionidae* are found large numbers of shells of *Vivipara* named by Sintzov *Vivipara novorossica*, the latter not at all differing from *Vivipara fuchsi* Pen. Among the shells of *Vivipara* we have found *Vivipara bifarcinata*.

5. The complex of fresh-water fauna of the southern areas of the Ukrainian SSR mentioned above are faunistically nearly identical with the complex of Lower Paleocene beds of Slavonia described by Neumayer (12).

6. The mode of occurrence of these beds in the south of the Ukrainian SSR and in Slavonia are similar, and these deposits far removed from each other are linked together by the Roumanian beds, analogous both faunistically and stratigraphically, located in the basin of the upper parts of the Buzeo and Ialomizy rivers.

7. The existence of a similar type of fresh-water horizon may be traced spreading from the southern regions of the Ukrainian SSR through Roumania to Slavonia and lying between Pontian deposits (analoga of the VII horizon of the Kamyshburun profile) and Maeotic deposits. The maximum thickness of this horizon in the south of the Ukrainian SSR coincides approximately with the latitude of the village of Grigorievka (10 km north of Odessa) and, according to our data, does not exceed 40 m lying below sea level. In their northern parts these deposits are thinning, until they pinch out and rise high above sea level.

8. The fresh-water series of beds considered is possibly a remnant of a vast fresh-water lake formed after the regression of the Maeotic Sea and stretched in a latitudinal direction, from the southern areas of the Ukrainian SSR to Slavonia; or a remnant of one or several river systems discharging into the receding Maeotic Sea basin and also oriented in a latitudinal direction. In the latter case it should be assumed that these river systems communicated in some way with one another. Confined to the fresh-water series of beds considered are also abundant occurrences of fossil vertebrates in the south of the Ukrainian SSR (N. Elizavetovka, Grebenniki, etc.).

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STRATIGRAPHY

APSHERONIAN FAUNA IN THE BASIN OF THE SEA OF AZOV

By V. V. BOGACHEV and V. P. EVSEEV

(Communicated by A. A. Borissiak, Member of the Academy, 28. X. 1939)

Up to the present the Apsheronian fauna has been regarded as cryptogeneous and limited in its range only to the Caspian basin. Not until the discovery on the Taman and Kerch peninsulas of Akchagyl deposits with Mactra fauna (I. M. Gubkin, N. B. Vassoevich, A. G. Eberzin) arose the question regarding the actual distribution of the Apsheronian fauna. One of the present authors (V. Bogachev) suggested the idea that it appeared in the Caspian basin as a successor of the Akchagyl fauna—from the west, moving from the Sea of Azov where it had formed from the Cimmerian fauna, and just in time to take advantage of the dispersal of the Akchagyl fauna for its own migration. The discovery by L. Sh. Davitashvili on the Abkhazian coast of the Black Sea of Upper Pliocene fauna containing *Cardiidae*, greatly resembling the Apsheronian ones, furnished evidence in support of this contention.

This is exactly how, in the author's opinion, *Didacna* of the Caspian type (*Didacna parvula*, *Did. catillus*), characteristic of the Baku stage, had emigrated from the Azov-Black Sea basin. The fauna of the Apsheronian stage contains no forms, from which the above fauna could have possibly descended.

The Akchagyl fauna is just as cryptogeneous. While N. I. Andrusov and many others believed it to have derived directly from the Sarmatian fauna, other authors assumed it had originated in the boreal region (S. A. Kovalevsky), or in the south (A. D. Arkhangelsky, V. P. Kolesnikov) and immigrated into the Caspian basin after the ultimate disappearance of the Pontian Sea.

However, so far we have not had any grounds for maintaining the view that the Apsheronian fauna which had directly replaced the Akchagyl one, had migrated from the Black and Azov Sea basins. An analysis of the peculiarities of Apsheronian *Cardiidae* speaks in favour of their descent from the Cimmerian fauna. At the present time we have received from the geologist G. I. Popov new material substantiating our view.

A new bore-hole in the town of Yeisk (at a depth of 102 m) was sunk through a series of coarse-grained quartz sands containing various pebbles—milky quartz, pure pellucid quartz, black and brown flint and coal fossils. However, in this case of greatest interest is the 37 to 49 m horizon where the sands are crowded with fresh-water shells. Predominant are: *Paludina* of the type *P.* (*viviparus*) *diluviana* with all the varieties. Particularly characteristic are the slender, turreted new forms which seem to present a further develop-

ment of var. *gracilis* Neum. But inasmuch as *Paludina diluviana* is a post-Pliocene form, ours should be regarded as an independently developed species (a new one). *Lithoglyphus* is met in great numbers, there is an abundance of *Dreissensia polymorpha* with its varieties, however particularly important are finds of *Unio sturi* M. H. and *Unio* (*Limnoscapha*) *maximus* Pen. non Fuchs.

This fauna, as will be seen later, has a wide range of distribution. *Unio sturi* and *Unio maximus* were revealed in Eastern Ciscaucasia together with typical Apsheron forms: *Apscheronia propinqua* and *Didacna intermedia*. As indicated above, thus far the range of the Apsheron fauna was believed to be confined to the Caspian basin, but now the Yeisk bore-hole yielded *Apscheronia propinqua* Eichw. at a depth of 41 m, alongside of the above-mentioned *Unionidae*. However, the coast of the Azov Sea was not the only place where Apsheron fauna was identified; in the Eastern Manych valley, in a bore-hole near the Divny village, were found *Apscheronia propinqua* and forms from the group *Didacna intermedia* as well (at a depth of 69.40 m).

Thus, the distribution of Apsheron deposits over the Manych valley and the Sea of Azov depression is an established fact; hence the suggestion that the migration of the Apscheronian fauna, *Cardiidae* of Pontian character, from their home, from west to east, becomes fully credible. It should be noted that *Unio sturi* is met in vast numbers in the Apsheron deposits of the Kura valley (Kodzhashen mountain range, near Minguichaur).

The occurrence of black flint with coal fossils in the sands near the town of Yeisk is noteworthy.

For the first time (in 1903) *Unio sturi* and *U. maximus* were found on the river Sald, near the Nesmiyanov farm, and it is just where in close proximity are seen exposures of sands with flint, coal fossils (*Fusulina*, *Schwagerina*), corals (*Spirifer*, *Athyris)* and others. These are the so-called Ergeni sands, to which fluvio-glacial origin is assigned.

They are developed along the Don river, extending southward as far as Taman, with *Spirifer*, *Unio sturi*, with bones of *Elephas meridionalis*, *Etasmotherium caucasicum*, etc.

On the Don, near the village Nagavskaya, sands with huge blocks of flints containing Upper Carboniferous fauna are overlain by clays and sands with *Unio procumbens*, *Unio beyrichi*, *Unio sibirensis*, *Unio bielzi*, etc. They indicate a well-defined stratigraphic horizon; the higher horizon is characterized by an abundance of *Paludina* varieties: *Paludina* (*vivipa-rus*) *diluviana* Rynth., and *Unionidae*, no more of the Levantine, but of the contemporary Eurasian type. This fauna corresponds to the strata of the Azov Sea coast (the Taganrog, *Paludina* beds of the Miussy liman). The fauna contained therein is already the normal Caspian one.

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GENETICS

ON SELECTING PAIRS IN CROSSING AND BREEDING WINTER HARDY WHEAT

By M. I. SALTYKOVSKY and E. S. SAPRYGUINA

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Studies on cold-resistance in winter and summer wheat at various developmental stages led us to a conclusion that frost-resistance in wheat is connected not only with the vernalization stage, as thought before, but with the light stage of development, too ([1], [2]). We suggested on the above ground ([3]) that when such varieties are selected for crossing which as much approach each other in frost-resistance as possible, but greatly differ in the duration of the corresponding developmental stages, forms which exceed the two parents in respect to frost-resistance may be obtained among the hybrid progeny of later generations.

Along with a study of the importance of the developmental stage for frost-resistance conducted on pure-line material, we have done some work on first generation hybrids, too. This enabled us to check on hybrids of later generations the conclusions reached immediately following the physiological experiments conducted on pure-line variety material.

Among hybrids at our disposal for the initial experiments on production of transgressive forms we have selected one, namely *milturum* 321 ♀ ×Kooperatorka ♂. The two varieties entering this combination were well familiar to us with regard both to their frost-resistance and stage characteristics. The summer wheat *milturum* 321 which had been bred at Omsk is distinguished by a very prolonged light stage, very much longer, indeed, than in the winter wheat Kooperatorka which had been originated at Odessa. Vernalization stage in *milturum* 321 is a very short one. Kooperatorka possesses the opposite stage characteristics. It has a long vernalization stage, about 40 or 45 days, while the light stage is considerably shorter than that in the summer component.

Milturum 321 is one of the most frost-resistant varieties among summer wheat. When sown in autumn at Saratov, both under field and laboratory conditions, it repeatedly proved to withstand a temperature of—12° and even—15°; however, it perished most often at a temperature of about—10°.

The winter wheat Kooperatorka assumes one of the last places with respect to frost-resistance among the varieties of winter wheat cultivated at present in the European part of the USSR. It is the chief defect of this good variety. In the majority of cases it is frozen to death in step-regions at a temperature of—16° or —17°. Fields of this variety are frequently deadly damaged even by considerably milder cold, especially after thaws in the later part of the

winter. Selection for frost-resistance was done among second-generation hybrids of this combination. The work was arranged on the following scheme. One hundred hybrid kernels and 50 kernels of the most frost-resistant component, i. e. of the Kooperatorka, were sown per lot. During the winter the lots were exposed to cold at various temperatures. Two lots, in which all the plants of Kooperatorka proved to be killed by frost and only 2—3 hybrid plants per lot survived, were left in the hothouse until the plants matured.

Repeated selection for frost-resistance was conducted among the third-generation hybrids. After this selection which was severe, when all the plants injured by frost died, the best individuals were transplanted into pots. There were 15 such plants. In order to make them develop well, they were kept for a long time in a cool well illuminated greenhouse. All the study, the fourth generation included, was done at the All-Union Institute for Breeding and Genetics (Odessa).

In the fourth hybrid generation the frost-resistance of the lines selected in the third generation was determined.

Frost-resistance varied even within a single variety at different parts of a plot. In the later half of the winter, after thaws, this variation may even increase. The main reason lies in the water microregime which can be hardly made uniform in the plots placed in the open. In order to grasp slight deviations in the frost-resistance of the hybrids from the parental varieties, we have sown the two parental varieties on the two sides of the row of the hybrids. In each plot there was thus the following sequence of material: 1) Kooperatorka, 2) hybrid, 3) *milturum* 321, 4) hybrid, 5) Kooperatorka, 6) hybrid, 7) *milturum* 321, 8) hybrid, 9) Kooperatorka.

At the beginning of the winter when the period of frost was established, all the plots were transferred to a cold cellar where a temperature from—5 to—7° was maintained throughout the winter. The material was treated by natural frost at injurious temperatures; prognosis of weather was made use of. In the experimental data presented below there are shown the lowest temperatures of the soil at the knots of tillering.

Regeneration of plants after treatment with frost took place in a little hothouse. The control plots were subjected to regeneration in part during the winter in the little hothouse, but mainly in the spring in the open. The surviving plants were recorded when all the plants injured by frost died. Estimation of the frost-resistance of the hybrid lines was done at the Bezenchuk Experiment Station in 1936/37.

Let us turn to the results of trials for frost-resistance contained in the table. A glance would suffice to become convinced in a greatly increased frost-resistance of all the eleven hybrid lines represented. In many cases the hybrid lines have shown from 80 to 100% of survival, whereas both parental varieties have died out completely. In the table are contained data on 11 hybrid lines planted in 3 groups of plots. These groups were formed in accordance with the amount of grains of the hybrid lines. The greatest number of determinations falls to group I. In the fourth group there was the lowest amount of grains available. Accordingly, only two determinations, at —16° and—18°, were made here. In these experiments a great transgression of the frost-resistance of hybrids was similarly found. Thus, for example, at—18° in four hybrid lines survived 90, 89, 80 and 100% of plants, whereas the more frost-resistant of the parental varieties—the winter wheat Kooperatorka—showed only 3% of survival after this frost-treatment.

All the work was planned for production of transgressions in frost-resistance. However, the results surpassed our hopes. There arose some fear whether there could not occur any mistake, for example, by admixture of hybrids with some more frost-resistant forms. The doubts have still increased

Frost-Resistance of the F_4 Hybrids *milturum* 321 ♀ × Kooperatorka ♂
(Percentage of the surviving plants after frost-treatment in 1936/37)

Hybrids and parental varieties	Control at −5°, −7° all the winter through	Frost-treatment				
		−16°	−18°	−19°	−19.5°	−20.5°
Group I						
Kooperatorka	100	100	10	0	0	0
Hybrid No. 1	100	100	90	80	78	11
Milturum 321	100	0	0	0	0	0
Hybrid No. 2	100	100	100	89	90	80
Kooperatorka	100	100	30	0	0	0
Hybrid No. 3	100	100	100	100	100	80
Milturum 321	100	0	0	0	0	0
Hybrid No. 4	000	100	90	100	90	80
Kooperatorka	100	100	20	0	0	0
Group II						
Kooperatorka	100	90	10			0
Hybrid No. 5	100	100	100			80
Milturum 321	100	0	0			0
Hybrid No. 6	100	100	100	No experiments		100
Kooperatorka	100	100	30			0
Hybrid No. 7	100	100	100	were made		60
Milturum 321	100	0	0			0
Hybrid No. 8	100	100	100			40
Kooperatorka	100	100	25			0
Group III						
Lutescens 329	100	100	100			100
Hybrid No. 9	100	100	90			70
Kooperatorka	100	100	20	No experiments		0
Hybrid No. 10	100	100	89	were made		50
Milturum 321	89	0	0			0
Hybrid No. 11	100	100	90			25
Kooperatorka	100	100	0			0

when it appeared that all the plants which survived after frost-treatment had exclusively white ears. One of the parents in our crosses, namely *milturum* 321, showed a very well expressed red coloration of ears. The red ear is notably a dominant character. The second hybrid generation has confirmed this. We have found in some of the hybrid lines in F_4 pink-shaded ears; however, this brought little satisfaction to us. Owing to doubts which were caused by the hybrid material, we have postponed for two years publication of the present note.

In 1937/38 and 1938/39 we have arranged extensive studies on transgressive variation in frost-resistance, winter leaf-habit and other physiological problems, using new crosses of the same combination and a number of others. We have analyzed in sequence the hybrids F_2, F_3 and F_4 in a significant number of combinations. Numerous new instances of increased frost-resistance were obtained in the above-described hybrid combination and in a number of new ones as well. It was established that there were no errors in the original experiments discussed in this paper. Even the doubts regarding the coloration of the ears had to be rejected, since forms with red ears appeared in the next generation in the progeny of some plants.

The theoretical assumptions made on the ground of physiological analysis of the significance of developmental stages of frost-resistance in wheat were thus verified.

For the first time the synthesis of frost-resistant forms was successfully achieved according to a plan indicated by physiological studies.

It should be especially emphasized that the number of F_2 plants was only 200. In further works of 1937—1939, on hybrids of various generations and of many combinations, there were established a number of facts which are of prime importance for breeding winter wheat. Thus, for example, it was proved that the form-producing process of the vernalization stage in wheat hybrids depends not only on vernalization stage, but on the duration of the light stage in the parental forms, too. It has been established that a number of hybrids in combinations obtained after our system proved resistant not only to frost, but to rot caused by high temperatures, too. A number of hybrids proved to be resistant to rot alone.

There were established a number of additional moments of importance for winter wheat breeding. New results on the selection of pairs for crossing will be published after the extensive experimental data are worked out.

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GENETICS

CYTOGENETIC STUDIES ON THE GENUS *NICOTIANA* AND THE ORIGIN OF *N. RUSTICA*

By F. A. FATALIZADE

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Studies performed during the last decade have shown that the cytogenetic method furnished secure grounds for judging of problems of phylogeny and introduced serious corrections into plant taxonomy. At the present time there are enough examples which demonstrate that the origin of distinct plant species in various taxonomic divisions can be cytogenetically ascertained.

Data on the origin of *N. rustica* ([1],[2]) were incomplete so that a number of problems remained obscure which made further study desirable.

The present work has begun in 1936 with the aim to clarify the problem of the origin of *N. rustica* on the basis of cytogenetic study of the interspecific hybrids within the group *paniculata* (after Goodspeed) as well as between this group and *N. rustica*. As a material for the present study served *N. rustica* (n=24), *N. paniculata* (n=12), *N. solanifolia* (n=12), *N. undulata* (n=12), *N. glauca* (n=12) and hybrids between these species.

Somatic chromosome numbers were determined on permanent preparations stained with iron-haematoxylin (after Heidenhain). Meiosis was studied in pollen mother-cells on permanent slides stained with gentian-violet as well as on aceto-carmin preparations. Meiosis was mainly studied beginning by the first metaphase. Both rootlets and flower buds were fixed after S. G. Navashin (10 : 4 : 1).

Owing to limitation in space, we shall leave out the details and present here only data on the chromosome numbers at first metaphase in pollen mother-cells in the hybrids studied (table).

As may be seen from the data on the hybrids *N. rustica* × *N. paniculata*, *N. rustica* × *N. undulata*, *N. rustica* × *N. solanifolia* and *N. rustica* × × (*N. paniculata* × *N. solanifolia*), the characteristic mode of conjugation in these hybrids is $12_{II}+12_{I}$, while the hybrids *N. paniculata* × *N. undulata*, as a rule, form no bivalents or produce but very few of them. It is known, further, from literary data ([3],[4]) that haploids of *N. rustica*, as a rule, do not form bivalents. These facts demonstrate that: a) 12 chromosomes of the haploid set of *N. rustica* are homologous to the chromosomes of *N. undulata*, and the remaining 12—to the chromosomes of *N. paniculata* and *N. solanifolia*; b) *N. rustica* is of amphidiploid origin, and its components were cytogenetically far remote species. In favour of this is the fact that hybrids between *N. rustica* and the tetraploid *N. paniculata* ([5]) which possess 48 chromosomes (24 from *N. rustica* and 24 from the tetraploid *N. paniculata*) proved completely sterile.

Hybrids *	Somatic chromosome numbers in hybrids	Number of chromosome plates	Chromosome number at the I metaphase															Total
			12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
N. paniculata × *N. solanifolia*	24		256	47	15	—	—	—	—	—	—	—	—	—	—	—	—	—	318
N. paniculata × (*N. paniculata* × *N. solanifolia*) a. (*N. paniculata* × *N. solanifolia*) × *N. paniculata* **	24		343	58	16	5	—	—	—	—	—	—	—	—	—	—	—	—	422
N. paniculata × *N. glauca*	24		—	—	—	—	—	—	—	3	16	14	42	85	167	—	—	—	327
(*N. paniculata* × *N. solanifolia*) × *N. glauca*	24		—	—	—	—	—	—	—	—	7	11	26	71	129	—	—	—	244
N. paniculata × *N. undulata*	24		—	—	—	—	—	—	—	3	14	17	29	16	19	—	—	—	98
N. undulata × *N. glauca*	24		—	—	—	—	—	—	—	—	—	4	13	17	26	—	—	—	60
N. rustica × *N. paniculata*	36		—	—	—	—	—	—	—	—	—	5	8	12	65	7	2	1	100
N. rustica × *N. undulata*	36		—	—	—	—	—	—	—	2	11	9	16	73	10	3	—		124
N. rustica × *N. solanifolia*	36		—	—	—	—	—	—	—	1	4	7	13	17	53	9	6	3	113
N. rustica × (*N. paniculata* × × *N. solanifolia*)	36		—	—	—	—	—	—	—	—	4	2	11	16	39	8	—	1	81

Did all the modern forms of *N. rustica* come from a single original amphidiploid or from several amphidiploids produced by intercrossing of various species? Two possibilities could be suggested: a) *N. rustica* is an amphidiploid hybrid between the ancestors of the modern *N. undulata* and *N. paniculata*, or b) the existing various forms of *N. rustica* are the products of two original amphidiploids—*N. undulata* × *N. paniculata* and *N. undulata* × ×*N. solanifolia*.

The first hypothesis and the first half of the second one are corroborated by the evidence produced by their author ([1],[2]) and by our data as well. Besides, S. A. Eghiz (unpublished) succeeded in proving finally this supposition by producing amphidiploids *N. paniculata* ×*N. undulata* which were normally fertile, strikingly similar to *N. rustica* v. *scabra* (wiht minor differences) and readily intercrossed with *N. rustica* with production of fertile offspring.

The existence of *N. rustica* v. *scabra*, of *N. paniculata* and *N. undulata* in wild state in South America (Bolivia and Peru)***, as well as morphological similarity of the hybrids *N. paniculata* ×*N. undulata* and of the respective amphidiploids to *N. scabra*, make it probable that the original amphidiploids were the ancestors of *N. rustica* v. *scabra*. It is quite natural to suppose that the various forms of *N. rustica* might arise from such an ancient amphidiploid in the course of subsequent evolution, especially owing to an active intervention of man.

As to the possible origin of *N. rustica* from two amphidiploids, it may be also regarded quite probable. It is true, the available evidence for the first hypothesis is far more abundant, but taking into account the cytological data presented here [as well as those by Goodspeed ([2])], the morphology of the species *N. undulata*, *N. paniculata*, *N. solanifolia*, *N. rustica* and of the hybrids *N. rustica* ×*N. solanifolia*, *N. rustica* ×(*N. paniculata* ×*N. solanifolia*), we cannot see reasons to reject the possibility that certain forms

* All the hybrids enumerated in the table (except for the two first) were sterile.
** Inasmuch as no difference between these two combinations of hybrids could be detected, data relating to them are summed up.
*** *N. solanifolia* was discovered in Chili.

of *N. rustica* could be derived also from amphidiploids which arose from crosses between the ancestors of the modern *N. undulata* and *N. solanifolia*.

Modilevsky and Dzjubenko (6), on the basis of the fact that the hybrids *N. paniculata* ×*N. glauca* do not produce bivalents, and hybrids *N. rustica* × ×*N. glauca* (7,8) and *N. rustica* ×*N. paniculata* form mainly $12_{II}+12_{I}$, have assumed that, out of the 24 chromosomes of the haploid set of *N. rustica*, 12 chromosomes are homologous with the chromosomes of *N. glauca*, while the remaining 12 chromosomes—with the chromosomes of *N. paniculata*. Proceeding only from cytological evidence, one may go still farther, assuming that *N. rustica* is an amphidiploid hybrid between *N. paniculata* and *N. glauca*.

Fgi. 1. From left to right: flowers of *N. rustica* v. *scabra*, those of *N. paniculata*, those of an amphidiploid *N. rustica* x *N. paniculata* and of the trigenomatic hybrid *rustica-paniculata-glauca*.

Sterility in the hybrids *N. rustica* ×*N. paniculata* was overcome by chromosome doubling (5), and the amphidiploids obtained were crossed with *N. glauca*; the only plant which resulted (grown this year) had 48 somatic chromosomes (24 from *N. rustica*+12 from *N. paniculata*+12 from *N. glauca*), i. e. proved to be a triple hybrid (Fig. 1). On the basis of the above hypothesis on the amphidiploid origin of *N. rustica* from *N. paniculata* ×*N. glauca*, the trigenomatic hybrid *rustica-paniculata-glauca* should be fully fertile: as a matter of fact, it proved to be completely sterile and, although there were many bivalents (from 17 to 23), its meiosis was highly disturbed.

The hypothesis on the origin of *N. rustica* from hybridization between *N. paniculata* and *N. glauca* should be therefore rejected.

Summing up we take it for probable that *N. rustica* had arisen from two original amphidiploids (*N. undulata* ×*N. paniculata* and *N. undulata* × ×*N. solanifolia*) which in their turn were produced by crosses between ancestors of the modern species. Naturally, the possibility of further hybridization between them is not excluded, since they should readily cross and produce fertile offspring.

My thanks are due to Prof. S. A. Eghiz for continuous advice during the course of the work.

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GENETICS

EFFECT OF HEAT UPON THE MUTATION RATE IN HOP (*HUMULUS LUPULUS* L.)

By I. N. HOLUBINSKY and N. I. HOLUBINSKAJA

(Communicated by N. I. Vavilov, Member of the Academy, 4. XI. 1939)

One can hardly overestimate the importance of the discovery by M. S. Navashin ([3,4,5,8,15]) of the increase of the mutation rate in seeds subjected to prolonged storage, especially when we take into account the considerably augmented effect due to increased temperature and moisture during the period of storage. In the latter case the process of «ageing» may be reduced down to a few days ([2,7,5,9,10]).

We will not consider here the consequences of this discovery* for revising rules of storing seed, terms of storage, etc. ([8,10]). We shall only point out here the extreme importance of the method in question for breeding vegetatively propagated plants. Any variation, if desirable from a practical point of view, may be fixed by vegetative propagation without taking recourse to sexual cycle. If such is the case, the practical breeder working on vegetatively propagated plants is far less interested in the behaviour of this or that mutant when sexually propagated.

The object of our work, the common hop (*Humulus Lupulus* L.), is just a vegetatively propagated plant; owing to its advantages which were explained above, it was selected to serve for verification of the significance of Navashin's discovery. Our experiments were started in 1937.

Fresh seeds obtained from open pollination of female plants belonging to a commercial population were used for the work. About 50 days prior to the normal date of sowing into hotbeds, i. e. at the end of January (29.I), the lot of seeds was divided into 23 lots, 200 seeds in each. One of the lots served as a control, the remaining ones having been subjected to heat at 60° for various terms, at normal air humidity and in an atmosphere saturated with water vapour (in exsiccators containing water). The duration of treatments varied from 12 hours to 60 days. After it became clear that the seeds completely lose germination power in saturated atmosphere as soon as within 12 hours, the terms were reduced, and four additional lots of seeds were treated from 1 to 10 hours.

Seeds of all the lots were germinated in pots at the same time (20.III) at the laboratory. All the individuals which survived were first transferred

* The increase in proportion of mutation after long storing of seeds, i. e. the fact of «ageing», was known from the literature prior the publication of Navashin's works ([13,16]), so that «ageing» of seeds was not «di-covered» by Navashin in the strict sense, which by the way he did not claim himself ([4]). However, Navashin was the first to offer the correct interpretation and to show the great practical importance of the phenomenon in question.

to a school, and then, in the spring of 1938, to their final place. During the whole season their behaviour was observed.

Out of the 27 experimental lots of seeds only 9 germinated: those treated at 60° from 12 hours to 20 days inclusive and those treated during 1 hour at 60° in an atmosphere saturated with water vapour. The remaining lots did not germinate at all. The strongest effect thus proved to result from treatments in an atmosphere saturated with water vapour. As early as in 24 hours (in part even within 12 hours) the seeds treated in such a way become kind of burnt (roasted), they lose their gloss, turn dull and finally change their normal colour for that typical of burnt seeds. On the other hand, similar

Fig. 1.

Fig. 2.

Fig. 1. Seedlings from seeds treated by heat at 60° during 15 days. The upper left seedling is from the control lot, the remaining exhibit abnormalities in development: death of the meristem of the root-tips, swelling, etc. About 80% of such seedlings were incapable of further development.

Fig. 2. Left—a control seedling, right—a seedling from a seed treated at 60° during 15 days.

phenomena (in a slight measure, too) were noticed in seeds heat-treated at a normal humidity only on the 25th day*.

As should be expected, the plants grown from these seeds differed to some extent (sometimes very strongly, indeed) from the control. The noted deviations were analogous to those observed in various plants by Navashin and his collaborators ([2–7,10–12,14,15]). Anomalies of germination and in behaviour of plants during their growth, as well as cytological abnormalities were observed during 10 days (in a slight degree); the effect proved to be especially strong after the 20-day treatment, the proportion of abnormalities attaining in this case 75%. Anomalies of development were most strongly expressed at the first stages of growth, i. e. at the moment of germination (Fig. 1). These abnormalities consisted mainly in various malformations of the root: death of the root meristem, thickening of the roots, occurrence of sutures, kind of shrinking,

* Similar phenomena were observed by us in seeds treated with increased humidity at the ordinary room temperature (18—20°); they were, naturally, much slighter and occurred later (about the 10th day of treatment).

etc-*. The majority of such seedlings (up to 80% from the lot of seeds treated during 15 days and nearly 100% in the lot treated 20 days) died owing to their inability of obtaining nutrition from the soil. The minority developed adventitious rootlets from the lower part of the hypocotyle; in such cases they continued growth, gradually recovered and somewhat approached normal appearance. However, the abnormalities did not disappear during further growth, and the majority of the plants continued to exhibit developmental aberrations (Fig. 2 and 3). Very often the growing point of the stem died, and one of the side-shoots continued the growth of the main stem. The leaflets, especially those of the axillary shoots, were malformed and of a shape entirely different from that in the control plant (Fig. 3). Such abnormalities were observed during all the period of growth.

Fig. 3. Individual (malformed) leaves of plants of the kind represented on the preceding figure (right).

Analogous deviations (though in a considerably smaller amount) were observed also in plants obtained from seeds heat-treated in saturated humidity during 1 hour, as well as after treatment of hop cuttings with a series of salt solutions of various concentrations ([1]).

Along with observations on the growth and developmental anomalies of the seedlings and plants resulting from them, cytological analyses were also conducted. We have fixed root-tips of young seedlings at the cotyledon stage as well as of further developing plants; the S. G. Navashin's chromo-acetic formalin

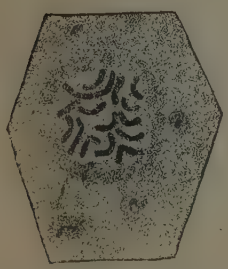

Fig. 4. Metaphase from the root-meristem showing chromosomes extruded into the cytoplasm.

Fig. 5. Anaphase showing lagging chromosomes and those extruded into the cytoplasm.

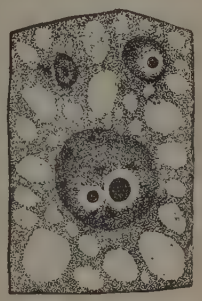

Fig. 6. Adventitious nuclei (micronuclei) due to chromosome extrusion into the cytoplasm, produced by abnormal division.

fixing fluid and staining with iron-haematoxylin of Heidenhain were used. Thus, not only viable plants were subjected to cytological investigation, but also seedlings which died at the initial stages of their development.

* Besides, it was noticed that the proportion of albino seedlings greatly exceeded that occurring normally. While in the ordinary cultures from normal seeds the proportion of albino individuals rarely exceed 5%, in the seedlings grown of seeds treated by heat it attained 14%.

Cytological studies have fully confirmed observations of earlier investigators ([2–4,6,9,10,12,14,15]) and failed to demonstrate something new on principle. We have found in our studies considerable deviations from the normal course of division, especially at the anaphase and subsequent stages. No full representation of such disturbances can be given as yet, owing to the fact that general knowledge of the karyology of hop is insufficient. Most striking were the instances of chromosome lagging on the spindle, irregular (uneven) disjunction of the chromosomes, extrusions into the cytoplasm and formation of the cells, etc. (Figs. 4—6). Finally, cells deviating in the number of haploid chromosome sets were frequent: haploid, tetraploid and other cells were met with. In adult plants (taken from the school in the summer) such disturbances of mitosis were considerably less frequent, but they nevertheless were found and in certain plants proved to be of constant occurrence.

In the spring of 1938 the plants were transplanted to the final place (although in somewhat crowded condition). Owing to the absence in the season of 1938 of the specialist (of the senior author of the present communication), observations on the experimental plants were reduced to a minimum, and a considerable proportion of them died; neither could be done cytological analysis of meiosis. However, even superficial observations have proved again developmental abnormalities as well as anomalies in the sexual sphere resulting in complete or partial sterility in spite of the normal development of hops. Besides, one of the plants strikingly differed from its sibs by extreme vigour, intense leaf coloration and energy of growth. All the external characters which remained the same in the summer of 1939 suggest the possibility of this plant being tetraploid. Cytological analysis should bring the ultimate decision.

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GENETICS

A CONTRIBUTION TO KARYOLOGICAL KNOWLEDGE OF MELONS

By A. G. ARARATIAN

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Karyological data on melons (*Cucumis melo* L.) are found mainly in the works by Kozhukhov ([2],[3],[4]). This writer has pointed out that chromosomes of melon are very small. He has observed polyploid cells in the meristem of the rootlets at the same time with the earliest students of this phenomenon in plants ([1],[10]). Finally, Kozhukhov has described in melons the, as yet not understood, persistence of the remnants of nucleolus in the cytoplasm ([3]). Data on the karyotype of melon are also found in Levitsky's work ([6]); from the number, size and shape of the chromosomes in melon and cucumber (*Cucumis sativus* L.) he concludes that the karyotype of melon with its 24 small chromosomes has probably arisen from that of cucumber (*Cucumis sativus* L.) with its 14 larger chromosomes. This idea is repeated in all the works by Kozhukhov cited above. Whitaker ([12]) also wrote on the karyotype of melon: from the study of mitosis and meiosis he supports Kozhukhov's conclusions on the chromosome numbers. Identical data are contained in Passmore's work ([11]).

In the present paper some data are advanced which were obtained by me in the course of a karyological study of a number of varieties of cultivated melons grown in Armenia and of the local wild melon (*Cucumis agrestis* Naud.). Material on the Chinese melon (*Cucumis chinensis* Pang.) was inspected by me, too. The material was contributed by S. Khachatrian, specialist in garden plant industry.

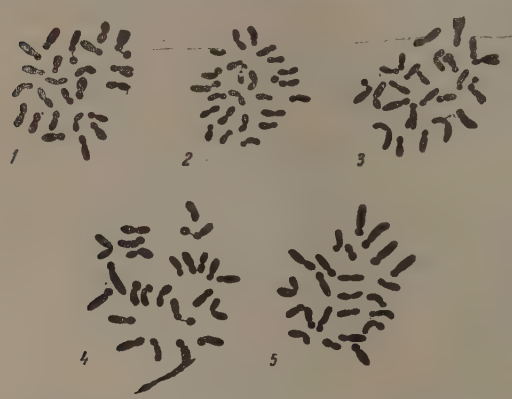
The following varieties of cultivated melons were subjected to investigation by me:

Cucumis eu-melo Pang., subspecies Adana, variety Shalakh
» » » » Khandaljak, unknown
» » » » Ameri, Sneivaz
» » » » Zard, varieties Miandab, Krmzi-ailiz, Daran, Krmzi-urjuk, Seid-Papakh, Kalamnash, Tarash
Cucumis microcarpus Pang., variety Shamam Turkestansky
» *flexuosus L.*, » Tarre
» *chihensis* Pang. unknown form

The following samples of the wild melons were studied by me (isolated by S. Khachatrian): Nos. 6, 16, 20, 26, 27 and 59. Root-tips of sproutlings were studied. Material was fixed with chromo-acetic-formol after Navashin and with chromo-formol (5.5) after Levitsky. Iron-haematoxylin was used for staining.

In respect to the number and size of the chromosomes my observations fully corroborate Kozhukhov's conclusions. In all the cultivated and wild forms of melons studied by me the somatic chromosome number proved to be 24. Chromosomes in various forms may be shown to differ in size; however, they are very small.

As to the chromosome shape, I have obtained a different result. Owing to application of chromo-formol and to over-differentiation of material fixed with chromo-acetic-formol (the most lightly stained chromosome plates were selected), I succeeded in demonstrating that all the chromosomes, the smallest included, are segmented. They consist of two arms each and are not simply bent rods obtuse at the one end, as described by Kozhukhov. Along with such favourable metaphases one might find in the same slides those with deeply-stained chromosomes, in which segmentation was scarcely visible or could be noticed only in some of them; the obtuse ends described by Kozhukhov could be seen here, too.



Nuclear plates of melon: *1*—var. Miandab, *2*—Khandaljak, *3*—Krmzi ailiz, *4*—wild melon, *5*—Chinese melon; ×3500.

Among the chromosomes of melon there are both nearly isobrachial and heterobrachial (chromosomes with knobs included) ones; the latter form predominates. There is also a pair of satellited chromosomes, as already described by Kozhukhov (3). It was very difficult in some cases, owing to the small size of the chromosomes, to place a given chromosome into the heterobrachial or knob-bearing group, when the short arm approached in shape a knob; it might appear therefore, that in some of the plates there was one pair of knob-bearing or heterobrachial chromosomes more. Sometimes uneven members of the former or latter might be thus counted. Nevertheless, all the chromosomes of melon may be classified into three groups; viz.: 1) nearly isobrachial and heterobrachial with the smaller arm exceeding a knob, 2) knob-bearing ones and 3) knob-bearing with satellites. There are 3—4 pairs of chromosomes of the first group; one of them is nearly isobrachial and sometimes rather large. Knob-bearing chromosomes are represented in melon by 8—7 pairs which differ from one another both in the size of the long arm and the short one, i. e. of the knob; there is just one pair of knob-bearing satellited chromosomes except for the Chinese melon where I failed to discover satellited chromosomes, probably owing to accidental circumstances.

In comparing karyotypes of cultivated varieties (Fig. 1—3) with that of the wild form (Fig. 4) we discover differences not only in dimensions, but in the shapes of homologous chromosomes, too. In the variety Khandaljak the chromosomes are smaller than in others (Fig. 2). In the wild melon and in the variety Krmzi-ailiz the pair of isobrachial chromosomes is well expressed and rather large (Fig. 3, 4), while it does not stand out in Miandab and Khandaljak. These data show that there exists karyotypic variation in melon, as also pointed out by Kozhukhov (2).

The karyotype of the Chinese melon differs from that of the ordinary melon, first, in containing a pair of small isobrachial chromosomes; secondly,

in a poor visibility of the satellited chromosomes. We cannot yet definitely state that the satellited chromosomes are lacking in general in the Chinese melon, as satellites often become discoloured before the rest of the chromosome and sometimes cannot be seen owing to their unsuitable position. Kozhukhov has seen satellites in the Chinese melon, too.

The fact that the chromosomes of melon are not merely simple bent rods, but represent dibrachial structures, like in many other plants where it was established, makes one to revise the hypothesis that the karyotype of melons has evolved from that of cucumbers. This hypothesis was founded on the assumption that the chromosomes of cucumber have dibrachial structure, while in melon this structure is lacking; the discovery of segmentation in the chromosomes of melon, too, leaves this hypothesis without a ground. Dibrachial structure of the chromosomes is a proof of their being complete structure, not mere halves. Characteristic differences in number and dimensions between the chromosomes of melon and cucumber cannot be regarded as direct evidence that the former has descended from the latter. Besides, if it be so, the two species should easily cross, which is not the case ([2],[12]).

The data presented above are against the assumption that cucumber and melon are two species closely related to each other and that the chromosomes of melon are derived from those of cucumber.

Kozhukhov in the one of his later works ([4]) has generalized his hypothesis of chromosome segmentation and has applied it to the whole family *Cucurbitaceae*; he thought that karyophylogeny in the family *Cucurbitaceae* has proceeded from small chromosome number to great one through chromosome segmentation. It appears highly dubious that karyophylogeny should be treated in isolation from the whole of the plant phylogeny and that it really proceeded in the family *Cucurbitaceae* through segmentation. At any rate the karyological knowledge in its present status does not support this assumption. The view common among cytologists that chromosomes might have arisen through segmentation has been criticized in the works by Levitsky and Tronn ([9]) and by Levitsky ([7]).

Tetraploid cells with paired and scattered chromosomes were very common in the root meristem in all the forms studied by me. This phenomenon is peculiar not only to the genus *Cucumis*, but to other genera of the fam. *Cucurbitaceae*, too ([2],[3]). There are other data showing that the phenomenon in question is common within entire taxonomic groups. Tetraploid cells and areas are found in spinachs both cultivated and wild which belong to distinct species. They are found in other genera of the fam. *Chenopodiaceae*, to which spinach belongs. The same phenomenon is common for *Hippophaë* and *Elaeagnus*, the two species of the family *Elaeagnaceae* (unpublished data of the present writer); it is clear that the tendency of producing tetraploid cells and areas in the root-meristem represents in certain cases a characteristic property of taxonomic groups—of genera and even families.

In my slides there were also very often present remnants of the nucleolus or nucleolus-like bodies which not only persisted «till the latest stages of the nuclear division» ([3]), but were found even at the interphase in the plasm, sometimes several of them of varying sizes. The remnants of the nucleolus assumed most varied position with respect to the division figure. In some cases a single body was found to lie in the middle of the plate or eccentrically. In other cases it was located to the one side of the plate; it had then often the shape of a drop with the thicker end turned toward the pole; or it was found at the one of the poles ([3]). Sometimes the nucleolus was found to elongate and to divide at the metaphase; its halves were driven to opposite poles ahead of the chromosomes. The remnants of the nucleolus were often found to lie outside of the spindle at various places within the cell. Such a behaviour

of the nucleolus was described in other plants, too. I have observed it in vine as well as in many plants which have been subjected to X-raying and to centrifuging.

I have observed in a single case non-disjunction of the chromosomes in a rootlet of the Chinese melon. All the chromosomes have split at late anaphase and the halves have disjoined except for a single chromosome; it has split, but remained lagging at the equator. A single case of non-disjunction and lagging was observed by me in vine. Rare instances of non-disjunction in somatic cells were described by Kostoff ([5]) and by others. This phenomenon is much more often in plants which have been subjected to various treatments, especially to X-raying ([8]).

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GENETICS

A NEW NARROW-LEAVED LARGE-SEEDED FORM OF *LALLEMANTIA IBERICA*

By D. J. VAKULIN

(Communicated by N. I. Vavilov, Member of the Academy, 4. XI. 1939)

While working up seed material of *Lallemantia iberica* F. et M. from experimental plants, we have found in 1938 a few seeds which differed rather strongly from the rest. These seeds were found in the sample No. 55 of pink-flowered *Lallemantia* which had been vernalized by low temperature two years in sequence (in 1937 at 2° during 10 days, in 1938 seeds collected from these vernalized plants were again vernalized at 2° during 30 days). We failed to find seeds of such sort in any other experiment or lots of white- or pink-flowered *Lallemantia*.

We have found 13 of such altered seeds; they were far from being identical; some of them were abnormally developed, as if grown together in two, the majority being of normal appearance and shape, but larger than usual. The great length of the seeds was especially apparent; they were from 1.5 to 2 times longer than the ordinary seeds of *Lallemantia*, whereas their width could be scarcely distinguished by eye from that of the ordinary seeds. The seeds made therefore an impression of being longer and thinner than usual.

All these seeds which were distinguished by their size were laid aside. As shown by our observations and experiment, the pink- and white-flowered *Lallemantia* flower and produce seeds under hothouse conditions (although not so well as the blue-flowering forms), so that we decided to sow some of these seeds the same autumn keeping the rest till the next spring. Observation on the plants grown in the hothouse together with the control has shown that they differed from the control in their larger dimensions and in having leaves which appeared at first sight to be more elongate.

In the spring 1939 the remaining 8 seeds were sown in the open ground side-by-side and at the same time with the control, i. e. with ordinary seeds of the same lot, in which the above-described modified seeds were found.

Sowing was done at the Odessa Botanical Garden on the 28 of March, 1939. Five seeds germinated; two of them unfortunately died and the remaining 3 individuals happily went on developing, bloomed and set fruit. Observations were done on them during the whole period of their life. The data relating to this question are summarized in Table 1.

As seen from Table 1, the three plants of the new form (No. 55/1, 55/2, 55/3) behaved differently from the control (ordinary form No. 55). First, the plants of the new form were somewhat later maturing, if one is allowed to speak of later maturity when the difference is only two days; however, this

Table 1

Forms	Sown	Germinated	First flower	Condition toward 14.VI	Harvest	Height in cm at maturity	Length in cm of the main fruit cluster	Number of whorls in the main cluster	Number of branches
New: 55/1	28.III	10—11.IV	6.VI	Blooms at the top	27.VI	49	25	11	10
55/2	28.III	10—11.IV	6.VI	Ditto	Yet immature	49	25	11	10
55/3	28.III	10—11.IV	6.VI	»	Ditto	51	29	12	10
Ordinary 55 (control)	28.III	8—9.IV	4.VI	Blossoming is over	25.VI	44	20	10	11.3

fact cannot be left out of consideration, inasmuch as all the phases of development were also shifted for 2 days, for instance sprouts of the new form appeared 2 days later, first blooming and the end of blooming was 2 days delayed as well. As to the harvest, one would say that one individual (55/1) matured 2 days later than the control, while the date of maturation of the remaining 2 plants was somewhat more delayed.

After harvest the plants were subjected to the usual laboratory analysis (measurements, calculations, weighing, etc.). As may be seen from the same Table 1, the difference between the new form and the control is rather clearly expressed here, too. The height of the new form is 5—7 cm greater than in the control (new form 49—51 cm, control—44 cm). The length of the main (central) fruit cluster is similarly greater by 5—9 cm (new form 25—29, control—20). The number of whorls on the main fruit cluster also proved to be greater in the new form; the number of branches was found to be somewhat smaller. Plants 55/3 and 55/2 differed especially in plus direction in respect to sizes.

Table 2 gives an idea of the absolute height and size of the seeds, as well as of the length and width of the leaves.

Table 2

Forms	Absolute weight of seeds in g	Dimensions of seeds in cm		Leaf size in cm	
		Length	Width	Length	Width
New: 55/1	5.0047	0.52—0.60	0.13—0.18	9.7—9.6	2.1
55/2	5.8220	0.60—0.65	0.18—0.20		
55/3	6.0.50	0.60—0.70	0.18—0.20		
Ordinary 55 (control)	4.3276	0.41—0.47	0.16—0.19	9.6	2.8

As to the absolute weight of the seeds (Table 2), a still greater difference between the new form and the ordinary one (the control) is observed. The weight of 1000 seeds of the new form exceeds that in control by about 1.5 g; this is a very perceptible difference.

No less perceptible and characteristic relations are seen in the dimensions of the seeds, especially the data on the measurements of their length; while the control gave a value of the order of 0.41—0.47, the new form gave 0.52—0.70 cm; the width of the seeds was less demonstrative. The deficiency of the seeds of the new form could be even seen by unaided eye; while comparing

seeds of the new form with the ordinary ones from the same sample (control), one could notice by means of a simple inspection that seeds of the new form (55/1, 55/2, 55/3) look larger and somewhat more elongated as compared with the ordinary control ones which appear shortened and smaller.

While inspecting young plants at the plantation, we have noticed that the leaves of the new form were somewhat different from those of the ordinary form; it was especially apparent in the case of the young apical leaves. The leaves of the new form appeared to be narrower and somewhat longer than those in the control. In lower leaves this character were less pronounced, but their width was invariably less than in the control. In Table 2 are presented data on the measurements of the leaves relative just to this case. Leaves from the 5th node were taken for the measurements.

On the base of the above clear distinguishing characters of the new form (55/1, 55/2, 55/3) we find it possible to distinguish it as a new variety under the name *Lallemantia iberica* F. et M. v. *angustifolia* D. Vakulin.

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GENETICS

CONTRIBUTION TO THE GENETICS OF COLOUR AND SPOTTING IN THE GOAT

By E. V. EYDRIGEVICH

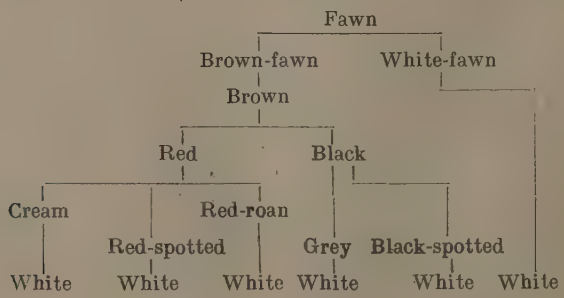
(Communicated by N. I. Vavilov, Member of the Academy, 17. X. 1939)

In animals cultivated for their fell or fur the colour of the hair-covering is a character of great economic importance which strongly influences the quality of the material produced. Different coat colours have a different value, according to the way they take colour when dyed, while the fur of spotted animals requires additional sorting.

The group of primitive Asiatic goats, including the Kirghiz goat, is characterized by a great diversity of coat colouring. Animals of this group show both variation of shades within a single colour and all kind of combinations of different colours. Of frequent occurrence are the following colours: white, grey, fawn, red, brown, black. Aside from these, there are spotted animals and animals showing a mixt colour.

F a w n. Is common for wild representatives of the species *Capra* and spread among primitive Asiatic goats. Fawn colour is characterized by a dark spine and a more intense colouring of the belly. Darker streaks are also bordering the head from above and below. The lateral parts of the trunk and head are of a red colour varying from dark-red shading into brown to solid white, but sometimes they are also of a grey colour of varying intensity. The coloured hair of the lateral parts of trunk and head is characterized by a zonal pigment distribution. Fawn was the original colour for white, black and all others which arose by way of mutation.

The diversity of coat colours in the Kirghiz goat, with reference to the trend of intensified or diluted pigmentation, may be diagramatically represented as follows:



Genetically fawn coat colour in all its variations is epistatic to black and hypostatic to white. When black aboriginal bucks are crossed with fawn does, the progeny proves to be fawn. When white Angora bucks are crossed with Kirghiz does, white progeny is obtained.

White. According to our observations on Kirghiz and Turkmen goats (1), as well as to data by Asdell and Smith (2) secured from British goats, and those by Lush (3) from Toggenburg goats, white is epistatic to the rest of colours. From crosses of pure bred white Angora bucks with does of most various coat colours exclusively white progeny was obtained. Mongrels Angora × Kirghiz have a solid white fleece and hair-covering of head and legs. In a small number of specimens thorough examination may reveal isolated coloured fibers along the spine and among the hair-covering. According to our observations, white kids are borne only in case at least one of the parents was white. No cases of white kids secured from coloured parents have been recorded by us.

White colour often involves various pigment patterns, most frequently black ones. In such cases usually not only the hair is involved, but also the skin area under the coloured hair. Black skin-spots are occasionally observed in the case of solid white coat colour. The distribution of white coat colour among various regions is far from being regular and is controlled by selection.

Black. In black-coated goats the fluff shows usually a more intense colouring than the barb. Black colour is often associated with grizzling. The latter varies from a scarcely noticeable admixture of white hairs to a complete shading into grey colour. In this case not only the fleece becomes grey, but also the hair-covering of head and legs. The goat seems to possess several genetically distinct types of grizzling. Black coat colour is also often accompanied with spotting on head and legs. The black coat colour is genetically hypostatic to the rest of colours. Crosses between black does and black bucks always yielded black kids, while we failed to get solid black progeny from crosses between black does and bucks of other colours.

Dappled Black. One of the varieties of black coat colour, characterized by a light colouring of the belly and light spots on head and legs, the rest of the animal's body being solid black. The colour of the belly varies from red to cream, shading into white. This coat colour is genetically epistatic with regard to solid black. With regard to the rest of colours it behaves in the same way as solid black, i. e. is hypostatic.

Grey. Varies from dark-grey to light-grey, shading into white. From the physiological point of view it is nothing else than one of the phases of the depigmentation process of black. This coat colour is genetically epistatic to black and hypostatic to white and fawn. Its relation to brown and red is not yet sufficiently clear; however, the occurrence of grey-red specimens whose fur consists of a mixture of black, red and white hairs, makes us assume that in this case we have to do with intermediate inheritance.

Brown. Varies from dark-brown, shading into black, to light-brown,

blending with red. Epistatic to black and hypostatic to white and fawn. Relation to other coat colours yet obscure.

R e d. Also varying widely from dark shading into cream. Epistatic to black and hypostatic to white and fawn. Relation to other colours still unclear.

S p o t t i n g s. There is a great variety of spotting in the goat. The nature of distribution of white areas in the most common type of spotting is diagramatically shown in Fig. 1.

In this type spotting first appears at the lower ridge of the right side in the form of a small white spot triangular in shape. Sometimes this is accompanied by the appearance of white spots on the extremities. The spot on the side grows, reaches the spine, spreading at the same time over the belly. In the subsequent pigmentation disappears altogether on the hind part of the trunk, to persist only on the fore part and the neck, later on only on the head, until it disappears altogether. The pigmented areas of the hair-covering in this type of spotting may show black, brown, red or any other colour.

Besides the type of trunk-spotting described above, there is another type of spotting in Kirghiz goats. It is characterized by rather small white spots scattered against solid black background all over the trunk of the animal. This kind of spotting is, however, of rather infrequent occurrence.

Quite common among the goat is spotting on the head which may also vary in expression. We suspend detailed description of this type of spotting, since it is lacking any economic importance.

According to our observations, spotting of the first type is dominating over solid colour. Crosses between spotted goats and those showing solid colour result in spotted progeny. Coloured areas are thereby inherited from the parent whose colour is the dominant one. Thus, by crossing a red buck with a black-spotted doe the progeny obtained will be red-spotted.

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PLANT BIOLOGY

MECHANISM OF BOLL DEHISCENCE AT THE TIME OF SEED SHEDDING IN *LALLEMANTIA IBERICA* F. ET M.

By D. J. VAKULIN

(Communicated by N. I. Vavilov, Member of the Academy, 4. XI. 1939)

In the majority of the field crops mature seed is notably shed at dry time; only a few plant forms show especial accomodations which preclude seed shedding at this time. *Lallemantia iberica* F. et M. belongs to this latter group. Unfortunately, it sheds great amount of seed at rainy weather; it is a most significant defect, which the workers dealing with *Lallemantia* are making efforts to eradicate. It was not without interest to study in some detail the structure of the seed-boll of *Lallemantia* and the mechanism of its dehiscence when moistened and shedding seed. We will be dealing with white- and pink-flowered *Lallemantia*.

Lallemantia has tubular 5-toothed calyx; the upper tooth, a kind of covering one, is broad, larger than the rest, oval in shape, slightly acute at the apex; two teeth on the two sides of it are narrower and more acute, the remaining two are narrower still, lanceolate and acute. At the bases of the teeth on the outer surface there are 5 small protuberances. They have never been mentioned in the literature. Along the calyx pass 15 nerves; each protuberance is located between two nerves and between neighbouring protuberances there is one nerve.

There are thus 3 nerves in each tooth; one of them is median, the two others running along the edge of the tooth. All three join at the acute apex of the tooth. The calyx is covered with rather coarse, short hairs which are scarcely visible to the nacked eye; they are more conspicuous on the nerves and at the edges of the teeth. The character of pubescence in the calyx is different from that in the teeth; the hairs on the former are gradually bent downwards, their base being nearly perpendicular to the surface or somewhat obliquely downwards; those on the latter are entirely straight and are directed upwards at an angle of nearly 45°. The teeth are pubescent both on the upper and lower surface and along the edges; the pubescence on the inner surface and on the edges of the teeth is somewhat longer. The upward direction of the trichomes of the teeth is obviously of importance for the outward movement of seeds from the calyx and aids shedding; the seeds slip out of the rather small opening over the hairs and are gradually pushed out of the calyx. The direction of the hairs obviously hinders retaining seeds within the calyx.

The peduncle of the boll is not long, but broad and flat, from 2½ to 3 times shorter than the calyx. The calyx is firmly attached to the plant and is thus not deciduous.

Each calyx contains 4 seeds, although not all of them are always normally developed; often only 2—3 grains are normally developed, the rest remaining underdeveloped. Mature seeds become separated from the mother-plant and «rattle» within the bolls when shaken. In dry time the calyx is closed from above with its teeth which come together so that seeds are not shed. On the other hand, in wet time the teeth bend outwards (straighten), open the mouth of the boll and seeds are shed.

The teeth are elastic in dry time; when artificially slightly bent outwards, they immediately revert to the original position. When moistened, the teeth become softer and although they straighten after having bent outward, they retain a certain elasticity. When moistened, the teeth of the calyx within a few seconds begin gradually bend outwards to open in this way the boll. An opening is thus formed which may assume a shape of trapezium, irregular pentagon or triangle, thus being fitted to seeds which are nearly triangular on cross-section.

In some cases the calyx is fully open; in others it is but incompletely opened, so that seeds cannot pass freely and stick in the mouth of the calyx between the teeth to form a kind of stopper; however, when the plant is shaken (by wind, etc.), the stuck grain is freed and falls out of the calyx; this is naturally aided in some measure by the hairs whose direction is favourable to the outward sliding of the seeds.

The teeth are unequally elastic: we are inclined to think that the lateral and the lower teeth are the ones most sensitive to moisture; elasticity is less pronounced in the uppermost large tooth.

The teeth when moistened work as follows: soon after moistening the lateral and lower teeth begin to straighten and move apart; this is one of the ways in which the opening is formed. The other was as follows: the film of water prevents for a certain length of time by its surface tension moving apart and straightening of the teeth, until forces making the teeth bend apart are in equilibrium with this tension or smaller than it. As soon as the teeth which gradually take the water in begin straighten with progressing energy, the tendency toward bending apart shows itself more and more, the power grows, so that the film which was holding them together is broken. The teeth become free and are sharply bent outwards, so that an opening in the calyx is produced at once by one step, in contrast to the previous case when it was formed gradually.

We think that all the teeth, except for the largest uppermost one, partake in breaking the water film. This tooth obviously partakes but little if at all in the formation of the opening in the moistened calyx. The uppermost tooth even hinders moving apart of the two neighbouring teeth, as they are covered by it; in any event, the edges of these teeth immediately adjoining it move apart slower than the opposite edges.

When only the uppermost broad tooth was artificially moistened, the calyx did not open at all or showed but slight traces of dehiscence: when, on the other hand, lateral teeth adjoining to, or rather covered by it, were moistened, they rapidly began their straightening and formation of the opening. Even when only one of the lateral teeth was moistened, it began to bend outward so that the calyx was slightly opened from the one side, while the rest remained in their usual position in close contact over the calyx.

When dry, the formerly moistened teeth bend inwards again, come into contact and close the mouth of the calyx even in case the seeds are already shed. Dry calyxes, if intact, are always closed. They open only for a short time when moistened; unfortunately, this short period of time, especially when there is a wind, is fatal, since seed losses due to shedding occur in this very period.

Taking into account all the above-said, breeding should be directed toward production of forms possessing calyx which does not open in wet time, whose seeds are firmly attached to the calyx when the latter is opened; finally, hairs in the mouth of the calyx should keep seeds from shedding when the calyx has partly opened.

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PLANT PHYSIOLOGY

PHASIC DEVELOPMENT IN VARIOUS BIOLOGICAL GROUPS OF *LATUCA SATIVA* VAR. *CAPITATA*

By F. E. REIMERS

(Communicated by A. A. Richter, Member of the Academy, 9. XI. 1939)

In the experiments reported in our previous paper (²) various varieties of cabbage lettuce were studied with regard to their phasic development. And it was found that within a variety grown on a natural daylength (latitude of Moscow) the plants were much alike in their morphological characters and sent forth floriferous shoots nearly at the same time. These results were to be expected indeed, since only choicest seeds had been taken to represent the varieties in our experiment. On curtailed daylength, however (10 and 14 hours), the plants within a given variety showed considerable difference as regards their morphological characters and time of ripening.

The experiments were continued in 1938, this time their purpose being to establish whether the length of the light stage observed in the case of a certain lettuce shrub, while growing it on a reduced daylength, would prove a hereditary character under the same conditions of development. If so, it should be possible, through selection of respective shrubs, to breed for a new variety differing in its behaviour from the initial one.

From this point of view a practical interest belongs to the spring and winter hothouse varieties of lettuce. In the practice of forcing these varieties formation of floriferous shoots is sometimes observed to start early in mass, and this depreciate their quality as a food article. Our experiments in 1937 have established that these varieties have a short vernalization phase which can go on at rather high temperatures. But their light stage is long, and it is just with regard to this character that the most considerable differences are observed between individual plants. We arrived therefore at a conclusion that the most simple method for avoiding early formation of floriferous shoots is to select plants according to the length of their light stage.

In the experiment of 1937 the differences between individual plants in the time of floriferous shoot-formation were most striking when they were grown on 10-hour daylength, but no ripe seeds could be gathered under these conditions, because the plants had not yet come to maturity when autumn set in. Only in the shrubs given 14-hour days did the seeds grow ripe.

In the middle zone of the USSR the lettuce is an obligate self-pollinizer. Taking advantage of this circumstance, we gathered the seeds separately

from each of the shrubs involved in the experiments of 1937. To obviate casual transfer of light seeds from one shrub to another, each of the shrubs was put under a big bag of gauze at the outset of flowering.

On May 13 1938 two thoroughly prepared beds received seeds from the following groups of lettuce, varieties Ideal and Eier Gelber.

Variety Ideal. No. 1. Various plants grown on natural daylength in 1937, whose progeny will be referred to hereafter as «initial variety». No. 2. The plant which in 1937 was the first among the 14-hour day plants to form a floriferous shoot—«short-phasic plant». No. 3. The plant which in 1937 was the last to start stem-formation among 14-hour day plants—«long-phasic plant».

Variety Eier Gelber. No. 4. Initial variety (see No. 1). No. 5. Short-phasic plant (see No. 2).

The plants grown in one of the beds were given natural daylength, those grown in the other, 14-hour daylength. The latter were shut out from light during the interval from 8.30 p. m. until 6.30 a. m. Both the natural and the short-day plants were divided into 2 identical sets, 15 plants in each. Observation was fixed upon each of the plants separately.

In that year as in the previous the plants on natural daylength were very distinctive by their form and size. No morphological differences between the plants of the initial variety and of the other strains was observed. But again there were striking differences between the 14-hour day plants within the initial variety. Many of them showed broad leaf rosettes spreading on the earth as in earlier experiments ([2]), while others were forming a normal and a quite tough head. Among the progeny of the other strains sorted out in 1937 all plants were of similar shape irrespective of the daylength they had received (Fig. 1).

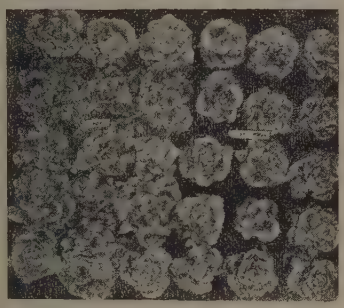

Fig. 1. Lettuce plants variety Ideal. The three right-hand rows belong to the «initial» variety. The three left-hand ones belong to a short-phasic strain selected by the method of phasic analysis.

Fig. 1 shows lettuce plants of the variety Ideal. The same phenomenon was observed in the variety Eier Gelber.

These data are sufficient ground for the conclusion that the morphology of plants is essentially determined by the conditions of their growth. No new statement, indeed.

But what is new about our results is that the plants which are outwardly quite alike when grown in certain surroundings refuse to change their morphology in a similar manner when transferred into another medium, but develop along various lines under new conditions. A thoroughly selected pure strain may have well preserved its varietal composition under certain conditions and still become strikingly diversified in morphology—a circumstance which should not be lost sight of when testing the strains in different localities and different years.

The dynamic in the formation of floriferous shoots among plants of initial varieties as well as short-phasic and long-phasic strains is brought out in a graph (Fig. 2). The curves show that when grown the next year on 14-hour or natural daylength, the plants from the seeds of short-phasic strains started and completed their stem-formation earlier than the plants of the initial variety, while the progeny of the long-phasic plant did this later.

The plants represented in Fig. 3 are short-phasic (No. 2) and long-phasic (No. 3) strains of variety Ideal grown on 14-hour daylength in a row on the same bed.

The difference between the two kinds of plants is so obvious that explanation of this figure would be superfluous.

In the initial varieties Ideal and Eier Gelber given 14-hour days the formation of floriferous shoots took more time (from the beginning of this process in the first plant till its completion in the last one) than it did in the progeny of short-phasic and long-phasic strains.

For the long summer day plants of the initial variety Ideal this process lasted 7—9 days. The same interval was engaged by shooting in the other strains of this variety when given 14-hour days. But if the plants of the «initial variety» Ideal were given 14-hour days, the formation of their floriferous shoots was drawn out over a period of 18 days. The same relations were observed among the plants of variety Eier Gelber. These data point to a high biological uniformity of the selected strains compared to the initial material, the variety itself.

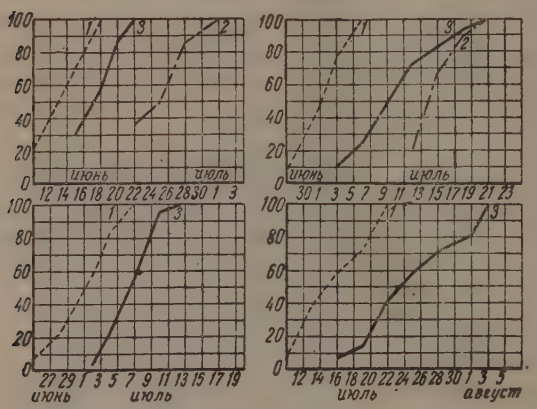

Fig. 2. Dynamics of formation of floriferous shoots in initial varieties and in selected strains given a different daylength: *1*—short-phasic strain, *2*—long-phasic strain, *3*—control (initial variety).

Fig. 3. The three left-hand rows—long-phasic strain, selected out of the variety Ideal. The three right-hand ones—a short-phasic strain

If conditions delaying the passage through the light phase are set up, the less uniform group of plants with regard to the length of that phase (initial variety) protracts the period of floriferous shooting in its various individuals to a greater extent than does the more uniform group (with regard to this character), selected by the method of phasic analysis. If, however, the conditions for passing through the light phase are favourable (long summer day), the difference between various groups of plants as regards the length of their light stage is smoothed out.

But even 7—9 days are too long an interval from the practical point of view. The range of variation in the time when individual plants come into floriferous shoot can, no doubt, be made shorter still by further selection of plants within the short-phasic and the long-phasic strain.

The number of plants in our experimental sets (50 and 30 plants per set in 1937 and 1938, respectively) was insufficient for selecting most valuable individuals from the varieties available, and they might have escaped our

test therefore. But the purpose of our work was to prove the possibility and even the necessity of applying the method of phasic analysis to selection of lettuce plants, as this had already been proved by T. D. Lyssenko (1) in the case of cereals.

We believe ourselves to have achieved this end.

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ANATOMIE DES PLANTES

SUR LA DÉTERMINATION DE LA POROSITÉ DU BOIS PAR MOYEN DES MESURES SOUS LE MICROSCOPE

Par ANDRÉ A. YATSENKO-KHMÉLEVSKY et NICOLAS N. BRÉGADZÉ

(Présenté par A. A. Richter, de l'Académie, le 9. XI. 1939)

Les années dernières, quelques investigateurs ont cherché par une série de mesures, effectuées sous le microscope, à caractériser quantitativement le bois pour en établir le diagnostic et évaluer les propriétés techniques. Dans ce but, un travail considérable a été entrepris par l'Association Internationale des Anatomistes du Bois, qui recommande à cet effet dans l'étude du bois de mesurer le nombre, la longueur et le diamètre des éléments de vaisseaux, la longueur des fibres, la hauteur et la largeur des rayons etc. ([1],[2]), et d'exprimer les résultats obtenus au moyen de termes standartisés spéciaux (peu, beaucoup, grand, petit, etc.), ayant un sens exact. Sans nier l'importance d'une telle standartisation des termes de dimension, il faut avouer que les données obtenues par ces mesures sont très incomplètes, surtout pour l'appréciation des qualités techniques du bois. Ces mesures ne nous donnent point le coefficient anatomique le plus important—la porosité du bois, c'est-à-dire le rapport des parois cellulaires à leur lumens (cavités). L'importance de ce coefficient est rendue claire par le fait constaté par plusieurs auteurs, que le poids spécifique de la matière ligneuse est à peu près le même pour toutes les essences et, d'après les déterminations différentes, est égal à 1.40—1.56 ([3]). Il en résulte qu'une des propriétés techniques du bois les plus importantes—son poids spécifique—dépend strictement du nombre de parois cellulaires contenu dans un volume donné du bois (ou de la porosité du bois).

La porosité du bois est le plus souvent déterminée par la méthode planimétrique, c'est-à-dire en mesurant la surface occupée par les cavités cellulaires avec un planimètre sur un dessin schématique de la coupe transversale du bois. Il en existent, en outre, autres méthodes plus directes, qui ne sont pourtant pas trop sûres ([3]) non plus.

On ne détermine la porosité du bois qu'assez rarement, cette tâche exigeant un travail considérable. Quand nous avons entrepris l'étude de la structure du bois des essences forestières caucasiennes, le problème d'une méthode simplifiée de la détermination de la porosité du bois s'est posé devant nous. La méthode la plus appropriée à nos buts, c'est la méthode dite linéaire, employée dans la pétrographie pour la détermination de la composition minéralogique des roches. Cette méthode, comme le démontre une série d'articles ([4]), donne des résultats excellents. La méthode linéaire ne peut donner plus que la méthode planimétrique dont elle est une dérivation sim-

plifiée. Il est évident qu'une identicité complète des résultats ne peut être obtenue que lorsque la distance entre les lignes, dont la surface de la roche est couverte, est infiniment petite. Donc, le but de l'investigateur est de chercher la distance maximum entre les positions de la ligne à mesurer donnant des résultats satisfaisants. Un travail de Nicolaiev (4), récemment paru, donne une analyse de la méthode linéaire, basée sur la théorie des probabilités, où l'auteur arrive à la conclusion que pour chaque type structural des roches, les distances entre les lignes (ou la longueur générale de la ligne) doivent être obtenues empiriquement*.

L'adaptation de la méthode linéaire est d'autant plus facile que dans le bois nous ne trouvons que deux composants (parois et cavités), et que leur distribution est assez régulière**. L'expérience des pétrographes nous montrant que le problème ne présente aucune difficulté au point de vue théorique, il fallait donc essayer la méthode linéaire pratiquement.

Dans notre travail, les questions suivantes étaient à éclaircir: 1) la dimension de la partie de la coupe à mesurer; 2) la distance entre les lignes de mesure; 3) la direction des lignes (radiale, tangentielle ou diagonale); 4) le nombre de mesures et 5) le temps nécessaire pour obtenir les résultats définitifs. Comme objet de notre étude nous avons choisi les bois à pores épars suivants: le hêtre, le peuplier, le buis et l'érable. La première étape de notre étude devait nous fixer sur la possibilité de substituer sur un dessin la détermination planimétrique de la surface des parois par une détermination linéaire. A ce but nous avons fait, à l'aide d'une chambre à dessin d'Abbé, des croquis des coupes transversales et tangentielles sur un papier quadrillé. Sur ces croquis, avec un planimètre, on déterminait le % de la surface occupée par les cavités des vaisseaux et des fibres et les rayons (sur une coupe tangentielle), après quoi le dessin était couvert d'une série de lignes parallèles à la limite de la couche annuelle, perpendiculaires à la limite et passant sous des angles différents. Sur ces lignes, nous avons mesuré les portions traversant les cavités et les parois, et les rayons et le reste du bois. Il a été impossible de choisir un grossissement du microscope avec lequel on pouvait dessiner exactement un nombre considérable de vaisseaux et les cavités des fibres, ce qui nous a obligé d'avoir, pour chaque essence étudiée, trois séries de croquis—de la coupe transversale à un faible grossissement, dans lequel seuls les contours des cavités de vaisseaux (pores) étaient dessinés, de la coupe transversale à un grossissement plus fort, où étaient figurées les cavités des fibres et des vaisseaux, et de la coupe tangentielle, où les contours des rayons médullaires étaient représentés.

Pour le bois de hêtre nous avons fait quelques dizaines de séries de croquis: pour chacune des autres essences étudiées—4—5 séries. Les résultats des mesures ont démontré que les chiffres obtenus sur un nombre de lignes suffisant sont très proches des chiffres obtenus à l'aide de la détermination directe avec le planimètre.

La seconde étape de notre travail était consacrée à des mesures faites directement sous le microscope à l'aide d'un oculaire à micromètre, sans faire de croquis sur papier. Les mesures étaient faites en comptant le nombre de traits de graduation de la règle occupés par les parois et les cavités ou, réciproquement, les rayons et la masse du bois.

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* Les temps derniers, la méthode linéaire dans la pétrographie est souvent remplacée par la «méthode des points» où le travail du calculateur est facilité par un appareil spécial—le pusch-intégrateur(5).

** Pratiquement, il faut distinguer dans le bois trois composants: les cavités des fibres et des vaisseaux, leurs parois et les rayons médullaires. Ces derniers diffèrent de tous les autres éléments du bois par leur direction, la présence des cellules parenchymateuses et la composition chimique de leurs parois. Les identifier avec la masse du bois—c'est commetre une faute grave.

Après avoir fait un grand nombre de mesures, nous avons établi que la méthode linéaire donne des résultats excellents qui ne diffèrent presque pas des résultats obtenus par la méthode planimétrique.

Pour obtenir les données sur la surface occupée par les cavités vasculaires, il est nécessaire de procéder aux mesures linéaires sur une couche annuelle entière, avec un grossissement de microscope de 50 à 250 fois (selon la largeur de la couche et la dimension des vaisseaux). La règle de l'oculaire à micromètre doit être placée dans une direction radiale. L'intervalle entre les différentes positions de la règle variaient entre 0.07 et 0.015 mm (la préparation microscopique était mue ou par le vis d'une table mouvante ou par une pression délicate du doigt).

La mesure des cavités des fibres (auxquelles nous ajoutions les cavités du parenchyme ligneux) était faite à un grossissement plus fort (×1050), avec l'intervalle entre les positions de la règle de 0.015 mm. Ces mesures étaient faites ou dans la partie moyenne de la couche, ou bien dans le bois final et dans le bois initial, après quoi on évaluait la moyenne. Les cavités vasculaires rencontrées n'étaient pas escomptées.

Le % de la surface occupée par les cavités des fibres était additionné au % de la surface occupée par les cavités vasculaires, et la somme indiquait la porosité du bois.

Les rayons médullaires étaient mesurés sur une coupe tangentielle*. La direction de la règle était perpendiculaire à la hauteur des rayons. Le grossissement était de 100 à 200 fois et la distance entre les positions de la règle de 0.17 mm. Les cavités cellulaires des rayons n'étaient pas prises en considération.

Le % de la substance ligneuse était obtenu par une soustraction de 100 du % de la masse totale des rayons et du % des cavités. Dans tous les cas afin de recevoir un seul chiffre, nous faisions de 30 à 50 mesures.

Essence	Largeur de la couche annuelle, mm	Méthode	%		
			de la substance	des rayons	des cavités
Hêtre		planimétrique	37.13	21.5	41.36
	0.73	des lignes sur le dessin	39.46	20.99	39.55
		linéaire	39.40	20.03	40.57
	0.72	»	40.43	20.03	39.54
	0.36	»	29.67	20.03	50.30
Peuplier		planimétrique	29.84	18.13	52.03
	4.4	des lignes sur le dessin	31.35	17.41	51.24
		linéaire	30.91	17.78	51.31
Buis		planimétrique	52.15	20.63	27.22
	0.96	des lignes sur le dessin	53.66	20.20	26.14
		linéaire	53.70	19.25	27.05
Érable		planimétrique	47.29	13.65	39.06
	1.17	des lignes sur le dessin	49.30	13.05	37.65
		linéaire	49.27	13.54	37.19

* La mesure sur une coupe transversale, donnant le même résultat, est plus difficile, c'ar les contours des rayons y sont moins distincts.

Après plusieurs exercises dans l'emploi des deux méthodes exposées, nous avons procédé à un chronométrage très détaillé de toutes les opérations nécessaires pour obtenir les résultats définitifs (y compris la confection des croquis par la méthode planimétrique). Le chronométrage a démontré qu'en moyenne une détermination de la porosité par la méthode planimétrique demande 12 heures et par la méthode linéaire 2 heures, c'est-à-dire la dernière méthode accélère le travail de 6 fois.

En conclusion nous donnons une table qui réunit les données obtenues par les trois méthodes employées (planimétrique, linéaire et auxiliaire—basée sur des lignes effectuées sur le croquis).

En examinant les données de notre table, on peut se convaincre de l'identicité des résultats obtenus par les deux méthodes étudiées. La précision de la méthode linéaire varie de 1 à 2 %.

Grâce à sa simplicité, la méthode décrite dans notre Note est digne d'être largement utilisée dans la pratique des laboratoires préposés à l'étude du bois. Les résultats que l'on en obtient sont très importants pour juger de la valeur technique du bois. Nos études suivantes sur les relations de la structure anatomique du bois à son poids spécifique, entreprises avec l'aide de la méthode linéaire, seront exposées dans une Note suivante.

Pour finir, nous devons exprimer notre gratitude à M-me T. Kazakhichvili et Mr. P. Topouria, pétrographes à Tbilissi, pour une information sur la littérature pétrographique et minéralogique.

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PHYTOPATHOLOGY

MOSAIC OF WINTER WHEAT

By V. K. ZAZHURILO and G. M. SITNIKOVA

(*Communicated by N. I. Vavilov, Member of the Academy, 4. XI. 1939*)

No great interval has elapsed since mosaic was discovered on winter wheat in USSR, and data on its spread and harm are therefore very meagre. The

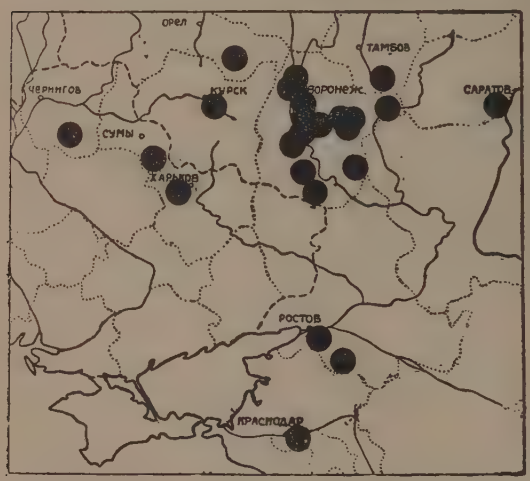

Fig. 1. Distribution of winter wheat in the USSR.

Fig. 2. Winter wheat *Hostianum* 0237. Left-hand plant healthy, the two right-hand ones badly affected by mosaic.

evidence produced within recent years is however sufficient to estimate the disease as very serious and deserving close examination.

Till now the mosaic of wheat has been recorded in the following provinces: Orel, Kursk, Tambov, Voronezh, Saratov, Sumsk, Chernigov, Kharkov, Rostov and Krasnodar* (Fig. 1). The most numerous are the reports from the Voronezh province where mosaic is attacking the winter wheat to a greater or lesser extent every year. In the years when the conditions for the disease

* Data referring to Ukrainian SSR are obtained from N. B. Broyakovsky, those on Krasnodar province, from P. G. Ruzinov.

to develop are especially favourable, the proportion of plants injured amounts in some areas of this province to 15—20%.

The first symptoms of winter wheat being affected with mosaic appear on the plants in autumn, 15—20 days after they have pushed above ground. The mosaic streaks are light green at first, but later, and especially in the next year, take on a more or less intense yellow colour. It is a veined kind of mosaic. In plants attacked by the disease growth is delayed and often stunted, but branching is more intensive. A rosette-like appearance (Fig. 2) is frequently the result of such a combination of symptoms.

The noxiousness of the disease is very considerable. In the observations carried on through a number of years in various regions of the Voronezh province the rosettes were found to perish without forming spike bearing stems, and the yield of mosaic plants was always strongly reduced (the loss ranging within 40—80%).

Anatomical and cytological modifications typical of virus diseases are observed in the affected plants: among them are necroses of the phloem, underdevelopment and reduction of plastids, nuclear hypertrophy attended by an increase in the number of nucleoli, formation of vacuolar inclusions within the cells (2).

Physiological modifications due to the attacks of mosaic are also pointing to the virus nature of this disease. Iodine test has shown that starch is retained in the light green portions of the leaves. By biochemical analysis it has been established that the diseased plants are higher in carbohydrates than the sound ones, but the amount of nitrogen is the same (Table 1).

Table 1

Plants	Analyzed in 1936	Analyzed in 1937	
		in abs. dry matter % of	
	% of starch in abs. dry matter	total nitrogen	soluble carbohydrates
Sound	2.82	2.73	14.24
Mosaic	4.97	2.76	17.13
«Rosettes»	—	2.84	26.22

Artificial infection with the juice from the diseased plants (conveyed to the leaves either by means of a needle or through rubbing them with emery dust soaked in the juice) was attempted without success. This will mean that the virus of wheat mosaic is not transmitted mechanically through the juice.

Wheat mosaic is very suggestive of the rosette-disease attacking winter wheats in USA. Wheat virus 1 (Mc Kinney) responsible for that disease is transmitted through soil. It was therefore only natural to suppose that in our case, too, the transmission of mosaic was by way of soil. To verify this supposition we started vegetation experiments in 1937 and 1938 with the soil from the Kamenno-Stepnaya breeding station plots, in which wheat plants were strongly affected with mosaic. Soil sterilized with formalin or virgin soil served as control. Sowing was done in all experimental sets simultaneously at a time most favourable to infection. The results proved negative. Beside these experiments sowing of winter wheat plants into the same soil sent by us was done in the same years at the Laboratory of Virus Diseases, All-Union Institute for Protection of Plants. Here the plants were not affected with mosaic either. It will be obvious that the disease is not

communicated through the soil. Neither is it transmitted by the vegetable remains, nor by the seed as it has been proved in special tests.

In order to test certain insects for their rôle in the transmission of mosaic to wheat, further experiments were carried on in 1939 by Sitnikova on the Mitrofanov experimental field. The experiment involved various species of plant lice as well as *Delphax striatella*, *Cicadula sexnotata* and *Deltocephalus striatus*. This time the experiment was crowned with success: *Deltocephalus striatus* L. was found to be the transmitter in question.

The technique of the experiment was as follows. The insects caught in the field were allowed to feed upon plants affected for 2—5 days. This took place under gauze insulators. The diseased plants were then replaced by sound ones (in dishes) which had also grown under insulators and attained the stage of 2—3 leaves. There were no insects under control insulators. The results of the experiment with *Deltocephalus striatus* are summarized in Table 2.

Table 2

No. of insulators	Experimental sets	Time	Number of insects	Total number of plants	Diseased	% of diseased
1	Rosettes	5.VI—5.VIII	300	60	56	93.3
2	Mosaic plants	29.V—19.VII	200	123	29	23.6
3	» »	22.VI—27.VII	400	41	32	78.0
4	Mosaic plants from insulator 2	31.VII—21.VIII	400	59	14	23.7
5	Without preliminary feeding of insects	18.VII—21.VIII	300	195	2	1.0
6	Controls	29.V—27.VII	—	187	—	—
7	Controls	18.VII—21.VIII	—	127	—	—

Estimation of the diseased plants in the experiment was checked by iodine-test for starch and microscopic examination of diseased and sound plants. In all of the tests the diseased plants were found to retain starch in their leaves, and inclusion bodies as well as necroses of phloem were also observed.

The incubation period proved to equal 15—18 days, and this was in good agreement with the above-mentioned data on the first appearance of the disease under field conditions.

As no sterile insects were available in control sets, we had to establish whether the symptoms of mosaic in the experiment were not the result of some injury caused by the insects themselves. The following facts refute this assumption: 1) Mosaic appeared on newly forming leaves, not on the first leaves which had been under direct influence of the insects. 2) Part of the plants remained sound in all experiments, though the great number of insects involved assured a nearly equal probability for the plants to be injured. 3) In the set without preliminary feeding of insects the proportion of affected plants was very low, although the number of insects was high: the only way to explain this is to suppose that the overwhelming majority of the insects did not carry the virus if not fed preliminarily upon diseased plants. 4) The injury caused by *Deltocephalus striatus* is of the same character as in the case of *Cicadula obscura*, and yet no diseased plants were observed in the experiment with this latter insect. The implication of all these facts is that mosaic on winter wheat is due to a certain virus and cannot possibly be regarded as an injury caused by the insect.

Many virus diseases at present are known in wheat. And in order to identify the mosaic spread in the European part of the USSR with some other kind of wheat mosaic we have to compare their principal symptoms and peculiar features (Table 3).

Table 3

	Wheat viruses (Mc Kinney, USA)			Green and yellow mosaic of wheat (Wada and Fukano, Japan)	Virus of «zakuklivanie» in cereals (Sukhov and Vovk)	Wheat mosaic (Zazhurilo and Sitnikova)
	Virus 1	Virus 3	Virus 2, 4 5, 6, 7			
Rosette symptoms	+	—	—	Dwarfs without enhanced branching	+	+
Carbohydrate retention in leaves	Probably none			—	+	+
Phloem necroses	Do not appear to form			—	+	+
X-bodies	+	Very seldom	?	+	+	+
Protein crystals	—	—	—	—	+	—
Transmission by juice . .	+	+	+	?	—	—
Transmission by soil . . .	+	+	—	+	—	—
Transmission by insects and length of incubation period	—	—	?	?	*Delphax striatella*, 7—9 days	*Deltocephalus striatus*, 15—18 days

From Table 3 it is apparent that the mosaic studied by us may be identified with none of the mosaics reported in wheat. It comes nearest to «zakuklivanie» of cereals, but the differences here are still essential: no protein crystals are found in the cells of mosaic plants. The carrier is different and the length of the incubation period deviates considerably. From these differences the authors are inclined to infer that the wheat mosaic recorded as yet only in the European part of the USSR is due to a virus which is quite independent of that responsible for «zakuklivanie».

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AGROPHYSIOLOGY

SOME EXPERIMENTS ON POTATO GRAFTING

By A. A. KUZMENKO

(Communicated by A. N. Bach, Member of the Academy, 31. X. 1939)

It is nearly 70 years now since grafting of various potato varieties upon each other was first attempted at the Botanical Gardens in Kiev. The hybrids obtained in those experiments showed a number of interesting characters. And the method of producing vegetable half-breeds by way of non-sexual crossing was found fit to be recommended for practical work(1).

In the present year various ways of grafting eyes upon tubers between different varieties of potato were tried by us under assistance of M. Y. Yakovenko at the new Botanical Gardens of the Ukrainian Academy of Sciences in Kiev. For the purpose we used pure strains of potato which had been reproduced at the All-Union Institute for Breeding and Genetics (Odessa) since 1935 by means of summer planting. The tubers of Epicure, Ella and Voltman varieties had been supplied for the test in early spring by A. M. Favorov, to whom thanks are due. In Kiev the tubers were subjected to vernalization and then used in grafting experiments. Each one of the varieties under test had all eyes removed from its tubers and replaced by eyes which were not its own.

The operation on the tubers was done in three ways: the cuts were wedge-shaped, segmented or cylindrical. The first two were made with a grafting-knife, the third, by means of a cork-drill, 12 mm in diameter. Best results were obtained by the latter method, whereby a small cylindrical piece 2—3 cm long was transferred into a respective hole prepared beforehand in the tuber. The graft thus effected took perfectly well; this was ascertained by unearthing and examination of the site of union. The proportion of successful graftings was less when the cut was a wedge or a segment.

Most promising results were obtained in crossing Voltman with Epicure. Crossing was done April 28 when the tubers had not yet sprouted. Eyes from Epicure were inserted into tubers of Voltman and vice versa. The own eyes of the stock were always removed carefully and the scion eyes introduced in their place. Then the tubers were planted into pots, whereby each pot filled with ordinary soil received only one tuber. The pots were kept in hot-beds until May 29 when the vegetative organs had reached their full size, and the plants were pricked out in the experimental plot for further development. The root-system was thereby thoroughly washed and the tubers examined to establish whether the union between the stock (tuber) and the scion (eye) was complete and no tuber's own sprouts were present. The eyes were always found to have perfectly grown in the tuber, if the cut had been made

with a cork-drill. The cylindrical piece thus obtained fitted well into the hole bored in the tuber preliminarily by means of the same drill. The picture revealed by cutting such tubers up was that of a scion and stock grown together and united by continuous ties formed by the tissues of the developing eye on their way into the tuber. These served to convey to the young plant the nutriment stored in the tuber of a foreign variety.

Fig. 1. Potato plants: variety Voltman (left) and variety Epicure grown on the root of Voltman (right).

The seedlings of variety Epicure after feeding in that way upon tubers of variety Voltman produced plants whose outward appearance, the structure and colour of stems and leaves were not to be distinguished during the period preceding flowering from the control plants of variety Voltman grown on the same plot (Fig. 1). On the other hand, the plants which resulted from

Fig. 2. Potato plants: variety Epicure (left) and variety Voltman grown on the root of Epicure (right).

growing the variety Epicure upon the root of variety Voltman were quite similar to the plants of the same variety grown on their own root (Fig. 2). By the development of the leaf lobes and the colour of leaves the young plants had repeated the type of the stock variety which supplied nutriment to them, and the branching of the shrub was also identical. And this in face of the fact that the varieties chosen for the experiment were strikingly different from each other as regards a number of characters. They differed in the structure of leaves and stems as well as in the form and colour of tubers and in the length of vegetation period.

It is only when flowering was about to set in that the grafted plants began in the structure of their leaves to show characters distinctive of the scion variety. In some of the specimens among our experimental plants these scion characters became more and more apparent alongside of the characters peculiar to the stock variety. And by the end of vegetation most of the plants had their above-ground organs developed in such a way that the characters of both the scion and stock were present, though the latter prevailed.

Harvesting was done September 26. The tubers of the experimental plants were of a mixed type showing the peculiar features of both varieties. 30 small tubers were gathered, for instance, in the case of a plant grown from Epicure eyes inserted into a Voltman tuber. Most of them presented characters of both varieties, but the type of Voltman distinguished by its pink or pale red colour predominated. Only 2 tubers of that number were white, a colour typical of variety Epicure, while 5 tubers were of a mixed type with a predominance of Epicure's characters. Only a few tubers were yielded by plants obtained from grafting the eyes of Voltman upon Epicure's root, but in all of them the characters of the stock gained the superiority.

There were many more plants obtained by growing Voltman upon Ella and vice versa and by growing Ella upon Epicure. They were invariably found to combine the characters of the stock and the scion both in the above-ground parts and tubers. And in the majority of cases the influence of the stock was stronger, too.

The tubers harvested in this experiment from the principal and most interesting combinations of grafts have been laid in store to be planted in future. Our purpose is to study the behaviour of the plants which will grow from them and to establish whether the characters acquired through grafting may be transmitted by heredity.

The results of our experiment go to confirm the concept of mentor advanced by I. V. Michurin. They are also consistent with the views expressed by T. D. Lyssenko a year ago([2]) as to the rôle of plastic nutrient substances in the formation of varietal characters. The method proposed by Lyssenko in that paper has already been applied with success by several authors. And our experiment appears to show that we can go further still in this direction by inserting non-sprouted eyes of a potato variety into a tuber of another variety which has its own eyes removed. The most convenient technique to secure the requisite physiological union between the stock and the scion in this case is to carry on the operation on tubers by means of a cork-drill or a special stamp.

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ZOOLOGY

VALUE OF CARBOHYDRATES IN THE NUTRITION OF THE IMAGINAL STAGE IN SOME *HOLOMETABOLA*

By I. KOZHANCHIKOV

(*Communicated by S. A. Zernov, Member of the Academy, 23. X. 1939*)

Nutrition of the imaginal stage in holometabolic insects shows an evolution from the *Coleoptera*, among which the imago of *Adephaga* and of some groups of *Polyphaga* (*Chrysomelidae*, *Curculionidae*, *Bruchidae*) feed on substances rich, besides many other components, both in proteins and carbohydrates, to groups of higher insects, in which nutrition of imago is reduced. Proteins still remain in the food of *Apioidea*, *Vespodea*, and of some *Diptera* (flower dust in the first, blood and products of protein hydrolysis in the second group). In most of *Lepidoptera* and in many species of *Diptera* which feed on the nectar alone, food consist almost exclusively of carbohydrates. In higher forms of *Holometabola* nutrition is either reduced to the consumption of water, as in many *Lepidoptera* (e. g. *Pyrausta nubilalis*), or else it is lacking altogether, as in the most specialized forms of *Holometabola* (number of *Lepidoptera*, e. g. *Liparidae*, *Saturniidae*, *Lemoniidae*, *Lasiocampidae*, in *Trichoptera*, and some other groups).

From the above it may be seen that carbohydrates play an important part in the nutrition of imago of *Holometabola*. The source of carbohydrate nutrition is chiefly the flower-nectar whose composition differs in various plants with regard both to water amount([8]) and ratio of chemical constituents. It usually consists of glucose, fructose, saccharose, aromatic compounds (cumarin, essential oils), dextrins, traces of proteins, oxalic, tartaric, and malic acids, and ash elements ([13],[2]).

It is of interest to find out the respective nutritional value of the nectar's components and the way they are assimilated by anthophilous insects. This refers in the first line to carbohydrates which make up the bulk of the nectar, except its water content. Specialization with regard to definite carbohydrates may point to the relation of various *Holometabola* to definite groups of nectariferous plants, as well as account for their preferential feeding. It should be pointed out that in nature there exist besides polytrophic anthophilous, as, for instance, the domestic honey-bee or *Bombus* species, many oligotrophic and monotrophic species of anthophilous insects (*Andrena*, *Hylemia*). Along with the question as to the nutritional specialization, the knowledge of the nutritional value of different sugars for the imago is of prime importance for the study of the fertility of insects.

The works available on carbohydrate nutrition in insects may be subdivided into two groups. The first comprises studies of the enzymes of the

digestive organs, while the second contains direct investigations of the nutritional value of carbohydrates. The first group is represented by numerous papers ([11],[3],[10],[7],[12]). These give, however, but a superficial and indirect judgment of the nutritional value of different carbohydrates. The second group is represented by a single article ([9]) dealing with the value of sugars, polyhydric alcohols and glucosids in the nutrition of the honey-bee, and by a preliminary note ([4]) which is at variance with the former.

For the present work we made use of two species—*Calliphora erythrocephala* Meig. (*Dip.*) and *Acronicta rumicis* L. (*Lep.*). The insects were bred in the laboratory for several generation, each species from a single female. After emerging the insects in the number of 20—35 pairs were kept on due diet; life-time, fertility and period of maturation were recorded. The temperature was kept for all series at 20° C; carbohydrates were given either in solutions, or dry, in the latter case additioned with pure water.

The results obtained for *A. rumicis* are given in Table 1. The best food proved to be glucose. A somewhat lower nutritional value was shown by fructose, raffinose, saccharose and mannit. The fertility proved to be higher in series fed with mannit and raffinose. Starch and glycerine are unfavourable, while lactose and oxalic acid have an injurious effect, resulting in a shorter life in females even than that recorded for starving moths. The results obtained are in good agreement with those secured from our earlier work on *Loxostege*, *Pyrausta* and *Agrotis*; neither do they contradict to the data available for adult *Lepidoptera*, the presence of amylase and invertase having been reported in the salivary gland and intestine, respectively.

Table 1

Nutritional Value of Different Carbohydrates and Related Compounds in the Food of Females *A. rumicis* L.

Diet	*n*	Life-time (days)	Fertility (number of eggs laid)
Starvation	18	10.0(4—13)	—
Water	15	11.2(6—16)	65(0—230)
Oxalic acid, 5%	10	9.0(7—10)	—
Glycerine, 20%	20	12.1(4—18)	27(0—273)
Mannit, 20%	18	16.3(6—22)	95(0—409)
Glucose, 20%	20	20.1(8—32)	260(0—632)
Fructose, 20%	21	17.0(11—20)	94(0—220)
Saccharose, 20%	19	16.8(5—24)	94(0—360)
Lactose, 20%	20	9.2(5—14)	9(0—99)
Raffinose, 20%	14	17.0(6—32)	237(0—638)
Starch, 3%	15	12.0(6—18)	28(0—215)

The experiments with *Calliphora* flies were run in two series. In the first series the flies were fed with carbohydrates or alcohols only, without any proteins. The flies of the second series were given the same substances additioned with proteins. The nutritional value of carbohydrates proved almost equal in both series (Table 2). In the absence of proteins the highest nutritional value is shown by glucose. Near equal as to their nutritive qualities, though considerably inferior to glucose, are fructose, raffinose, saccharose and mannit. Still worse results were obtained with starch, while lactose and glycerine proved almost unsuitable as food. In case proteins are added, fructose shows almost the same nutritional value as glucose, whereas mannit

proves considerably less nutritive than saccharose and raffinose. Feeding on lactose gives the same life-time values as protein-defficient diet, while those obtained with starch and especially with glycerine are much lower. Flies feeding on proteins alone show quite a negligible life-time and total sterility.

Table 2

Nutritional Value of Different Carbohydrates and Some Other Substances in the Food of Females of *C. erythrocephala* Meig.

Diet	*n*	Life-time (days)	Fertility (number of eggs laid)	Diet	*n*	Life-time (days)	Fertility (number of eggs laid)
I series, without protein				II series, with protein			
Starvation	35	2.6(2—3)	—	Starvation	35	2.6(2—3)	—
Water	35	3.7(3—5)	—	Protein+water .	25	5.0(4—7)	—
Glycerine, 25% .	35	5.0(2—8)	—	Glycerine+protein	35	3.9(2—5)	—
Mannit (dry) . .	34	20.0(4—50)	—	Mannit + »	31	24.0(4—50)	15(0—127)
Glucose (dry) . .	35	28.3(5—50)	—	Glucose + »	35	50.3(10—77)	194(0—556)
Fructose (dry) . .	34	19.0(8—29)	—	Fructose + »	35	56.4(8—105)	120(0—302)
Saccharose (dry) .	34	19.2(6—36)	—	Saccharose+ »	34	41.0(2—83)	34(0—320)
Lactose (dry) . .	35	4.5(3—9)	—	Lactose + »	34	4.5(3—6)	—
Raffinose (dry) .	34	21.0(3—51)	—	Raffinose + »	35	37.7(6—65)	62(0—148)
Starch, 1% . . .	35	8.0(4—16)	—	Starch, 1% . . .	34	5.7(7—10)	—

The data obtained for *Calliphora* are also in rather good agreement with the results of the studies which point to the presence of enzymes. Those found in the digestive organs of these flies are amylase, maltase, invertase and raffinase ([11]). However, in this case, as well as in the experiments with *A. rumicis*, the nutritional value of starch proves quite insignificant, notwithstanding the addition of amylase and maltase. Different sugars seem also far from being equivalent as to their nutritive qualities in spite of the presence of enzymes.

Notwithstanding the taxonomic remoteness of both groups investigated, the results obtained for each of them are rather similar, except that moths display a greater specialization for glucose, while flies make scarcely any difference between fructose and glucose. Comparing our data with those obtained by Philipps([9]) for the honey-bee, we find an analogy with these insects, too: both lactose and starch prove unfit for them as food in spite of the presence of lactase, amylase and maltase in their digestive organs. Bees are, however, more strictly specialized to the assimilation of carbohydrates and related compounds, since they do not assimilate raffinose and mannit, contrary to what was found for polytrophic anthophilous among *Lepidoptera* and flies.

The data obtained permit of the following conclusion. Besides the complexity of the molecular structure of carbohydrates and the presence of respective enzymes, the details in the structure of sugars play an important part in the way they are assimilated by adult *Holometabola*. A comparison of the nutritional values of different carbohydrates and related substances shows that the more a given substance deviates from the glucose, the lower its nutritional value. This relation is made evident by comparing the results for glucose, fructose, mannit and glycerine. It should be taken to mean that nutritional specialization in anthophilous insects may be accounted for, besi-

des the peculiarities in morphological structure of both insects and flowers, also by the chemical composition of the nectar contained in different groups of plants.

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